

# Coil Design for Wireless Power Transfer

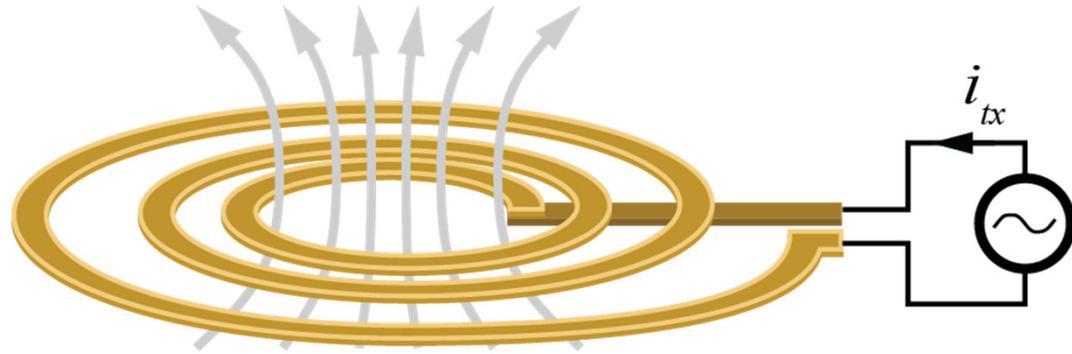
Daniel Costinett

With contributions from  
Ruiyang Qin, Jie Li

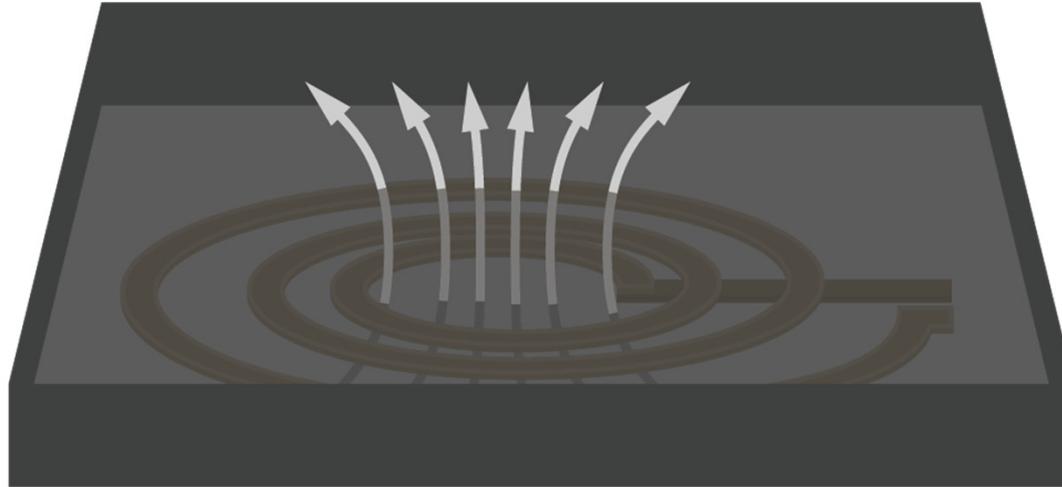


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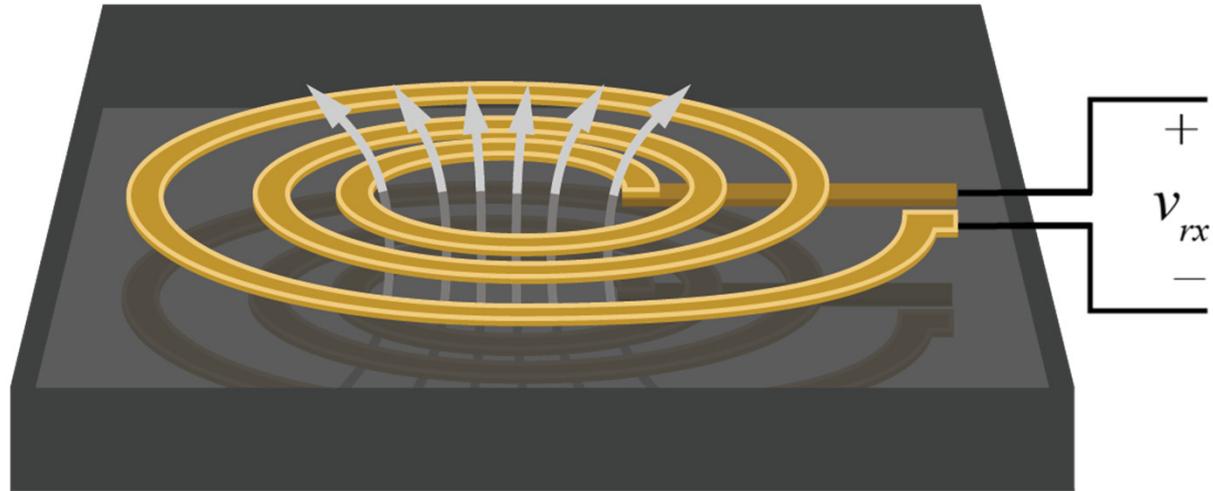
# Wireless Power Transfer (WPT)



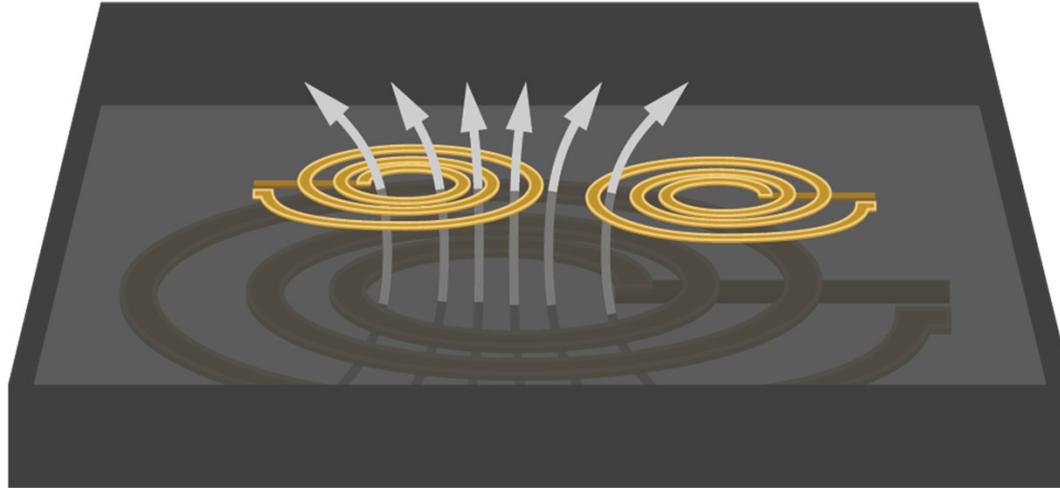
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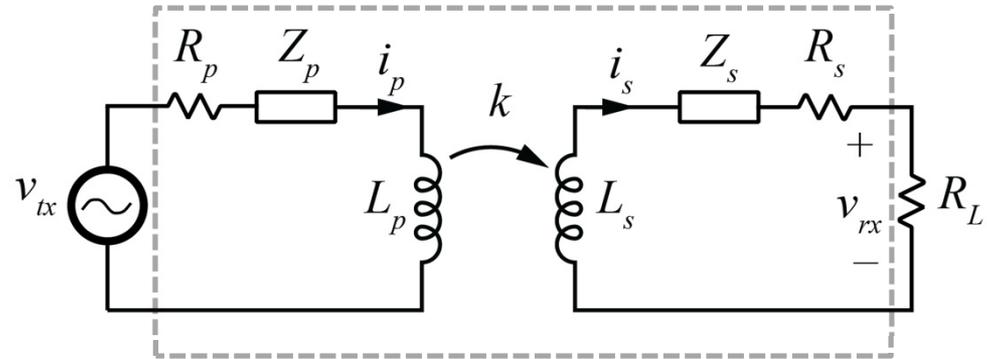
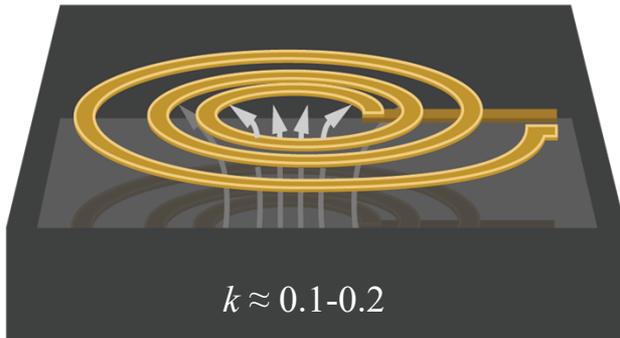
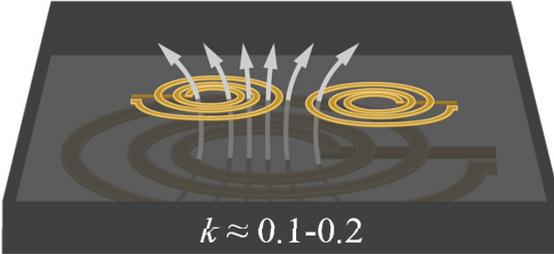
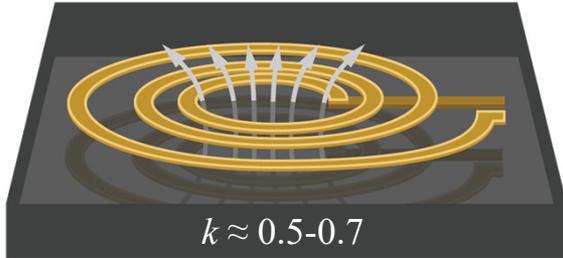
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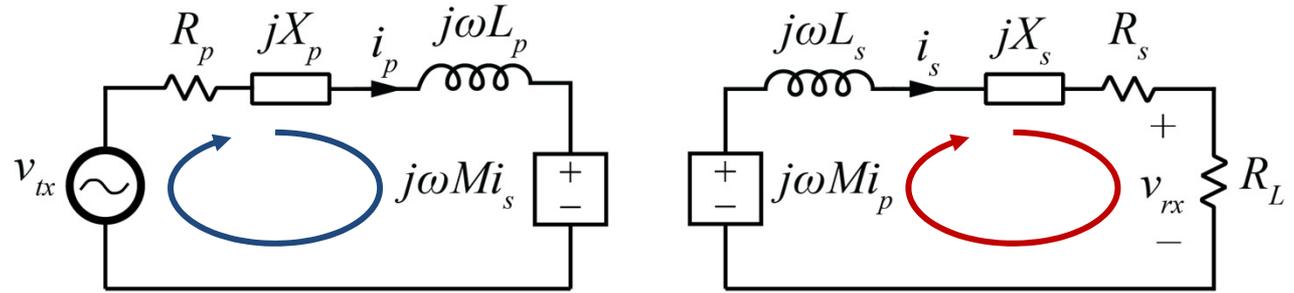
# Modeling Inductive Links



$$k = \frac{M}{\sqrt{L_p L_s}}$$

- To limit EMI & ac coil losses,  $i_p$  and  $i_s$  should be dominantly single-frequency ( $\omega$ )
- $Z_p$  and  $Z_s$  represent any additional reactance added or generated from Tx/Rx implementation

# Modeling Inductive Links



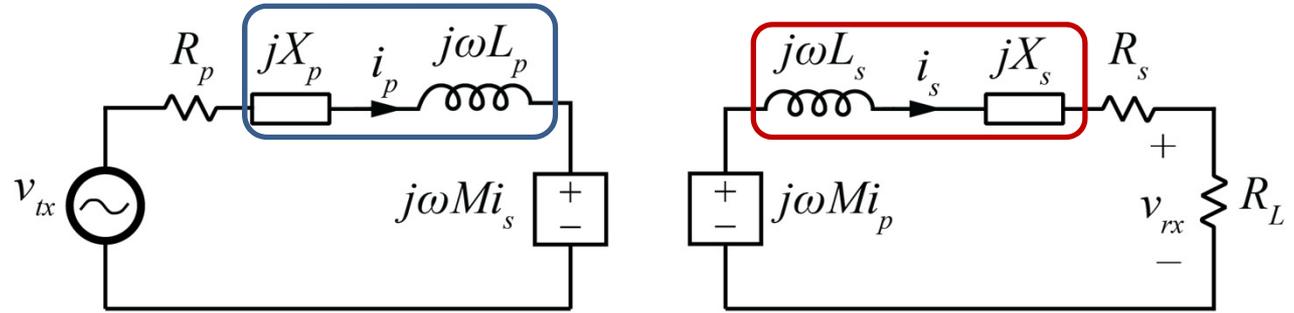
$$v_{tx} = i_p(j\omega L_p + jX_p + R_p) + i_s(j\omega M)$$

$$v_{rx} = i_s R_L = -i_s(j\omega L_s + jX_s + R_s) + i_p(j\omega M)$$

- With resistive load, average  $P_{out} = \frac{1}{2}|i_s|^2 R_L$
- The secondary current  $i_s$  results from the biasing source  $j\omega M i_p$ , where

$$\frac{i_p}{i_s} = \frac{R_L + j\omega L_s + jX_s + R_s}{j\omega M}$$

# Modeling Inductive Links



$$v_{tx} = i_p (j\omega L_p + jX_p) + R_p i_p + i_s (j\omega M)$$

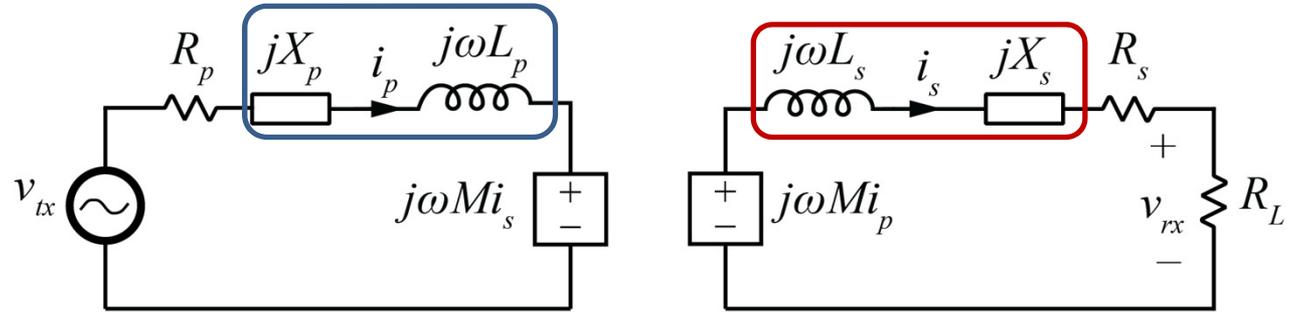
$$v_{rx} = i_s R_L = -i_s (j\omega L_s + jX_s) + R_s i_s + i_p (j\omega M)$$

- With resistive load,  $P_{out} = \frac{1}{2} |i_s|^2 R_L$
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$$\frac{i_p}{i_s} = \frac{R_L + j\omega L_s + jX_s + R_s}{j\omega M}$$

- Any **net reactance on the secondary** will increase primary current at the same power, increasing losses on  $R_p$
- Any **net reactance on the primary** will not affect loss, but will require increased amplitude of  $v_{tx}$

# Modeling Inductive Links



$$v_{tx} = i_p (j\omega L_p + jX_p) + R_p i_p + i_s (j\omega M)$$

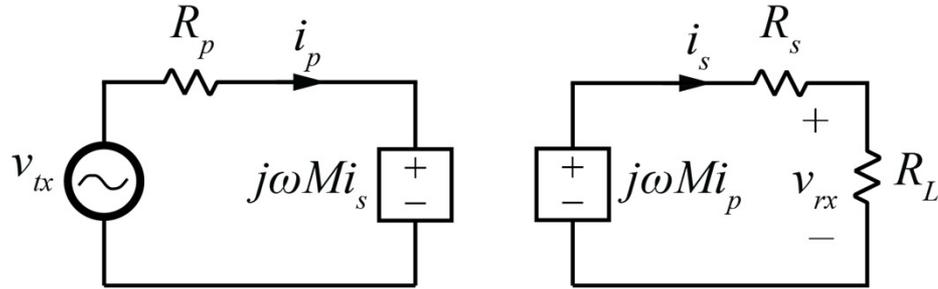
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- Any **net reactance on the secondary** will increase primary current at the same power, increasing losses on  $R_p$
- Any **net reactance on the primary** will not affect loss
  - Set  $X_i = -\omega L_i$
  - Control, unloaded, short circuit operation require additional consideration

# Modeling Inductive Links



$$v_{tx} = i_p(R_p) + i_s(j\omega M)$$

$$v_{rx} = i_s R_L = -i_s(R_s) + i_p(j\omega M)$$

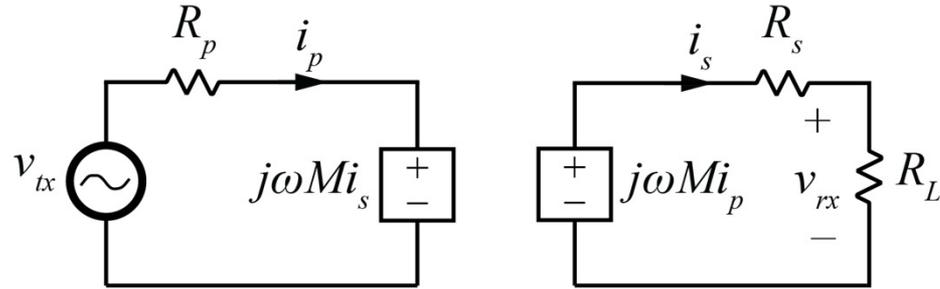
With series-resonance of both coils at  $\omega$

$$\frac{i_p}{i_s} = \frac{R_L + R_s}{j\omega M}$$

The power transfer efficiency is

$$\eta = \frac{\frac{1}{2} |i_s|^2 R_L}{\frac{1}{2} |i_s|^2 R_L + \left( \frac{1}{2} |i_s|^2 R_s + \frac{1}{2} |i_p|^2 R_p \right)} = \frac{1}{1 + \frac{R_s}{R_L} + \frac{|i_p|^2 R_p}{|i_s|^2 R_L}} = \frac{1}{1 + \frac{R_p(R_L + R_s)}{\omega^2 M^2}} \left( \frac{R_L}{R_L + R_s} \right)$$

# Modeling Inductive Links



To maximize efficiency, find the matched  $R_L$

$$\frac{\partial \eta}{\partial R_L} = 0 \rightarrow R_{L,opt} = R_s \sqrt{1 + \frac{\omega^2 M^2}{R_p R_s}}$$

which results in maximum efficiency

$$\eta_{max,R_L} = \frac{\frac{\omega^2 M^2}{R_p R_s}}{\left(1 + \sqrt{1 + \frac{\omega^2 M^2}{R_p R_s}}\right)^2} = \frac{k^2 Q_p Q_s}{\left(1 + \sqrt{1 + k^2 Q_p Q_s}\right)^2}$$

where

$$Q_i = \frac{\omega L_i}{R_i}$$

$$k = \frac{M}{\sqrt{L_p L_s}}$$

# Model Conclusions

$$\eta_{max,R_L} = \frac{k^2 Q_p Q_s}{\left(1 + \sqrt{1 + k^2 Q_p Q_s}\right)^2}$$

- To increase inductive link maximum efficiency, increase  $k^2 Q_p Q_s$   
 $-k^2 Q_p Q_s = 10,000$  for  $\eta_{max,R_L} = 98\%$
- Valid for  $X_s = -\omega L_s$  and  $R_L = R_{L,opt} = R_s \sqrt{1 + \frac{\omega^2 M^2}{R_p R_s}}$ 
  - Requires varying receiver impedance (magnitude and phase)
  - Difficult to do across varying  $k$  and with nonideal rectifier
- $\eta_{max,R_L}$  is the fundamental limit before considering any implementation of the power electronics

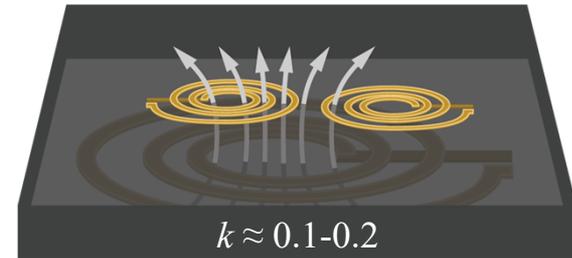
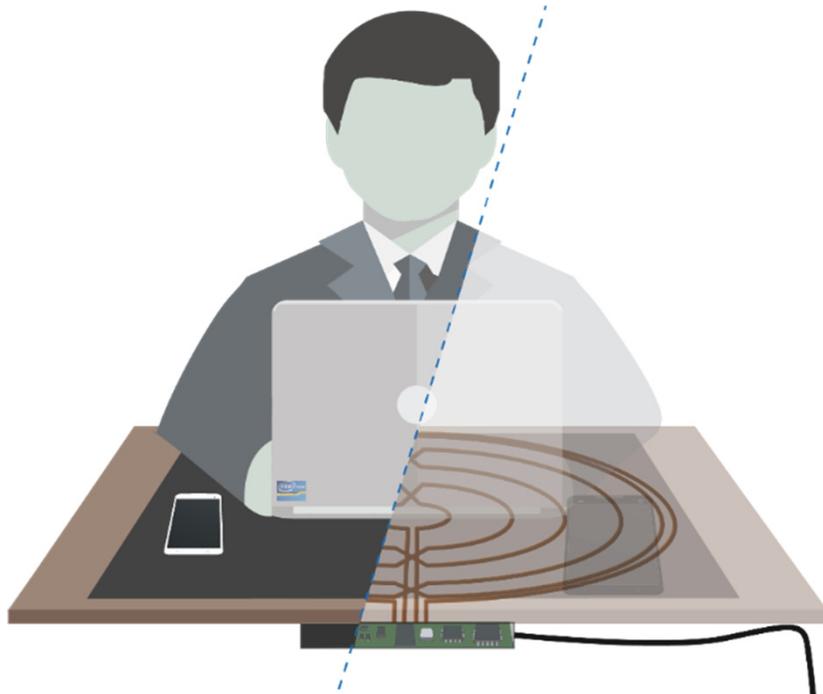
# Multi-Receiver Efficiency

- For  $N_R$  identical receivers with no cross-coupling

$$\eta_{max} = \frac{N_R k^2 Q_1 Q_2}{\left(1 + \sqrt{1 + N_R k^2 Q_1 Q_2}\right)^2}$$

- Efficiency increases with multiple receivers
- Requires identical coupling and each receiver to regulate resistance to a global optimum

# Example Application Design



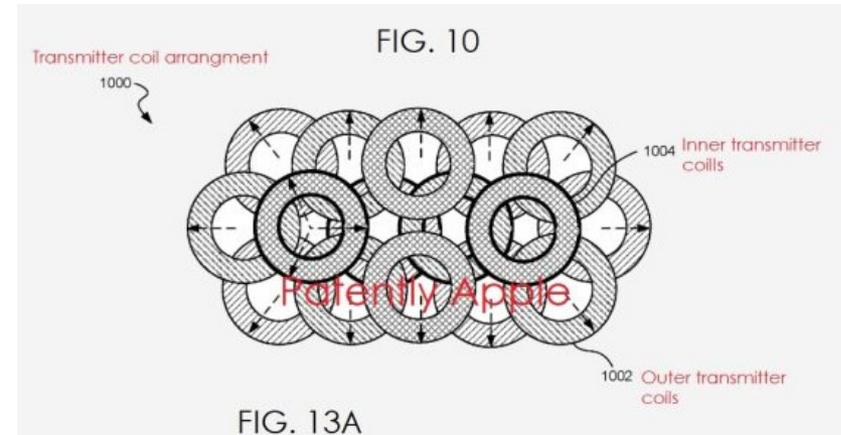
# Resonant vs. Inductive WPT

	Inductive single-coil	Inductive multi-coil	Magnetic resonance
Standard	Qi inductive 110-205 kHz		Resonant AirFuel 6.78 MHz
Positioning of receiver application	Exact positioning	Positioning more flexible (X and Y direction)	Free positioning (up to >30 mm vertical freedom)
Number of devices charged	Charges only one device	Charges one device but with better user experience	Charges multiple devices
Rx-Tx communication	In-band communication		Bluetooth low energy or in-band communication

Infineon, "Wireless Charging", <https://www.infineon.com/cms/en/applications/power-supplies/wireless-charging/>

# Increasing $k$

- Common approaches
  - Spatial constraints / enforced alignment
  - Multi-coil transmitter
  - Coil and core geometrical design



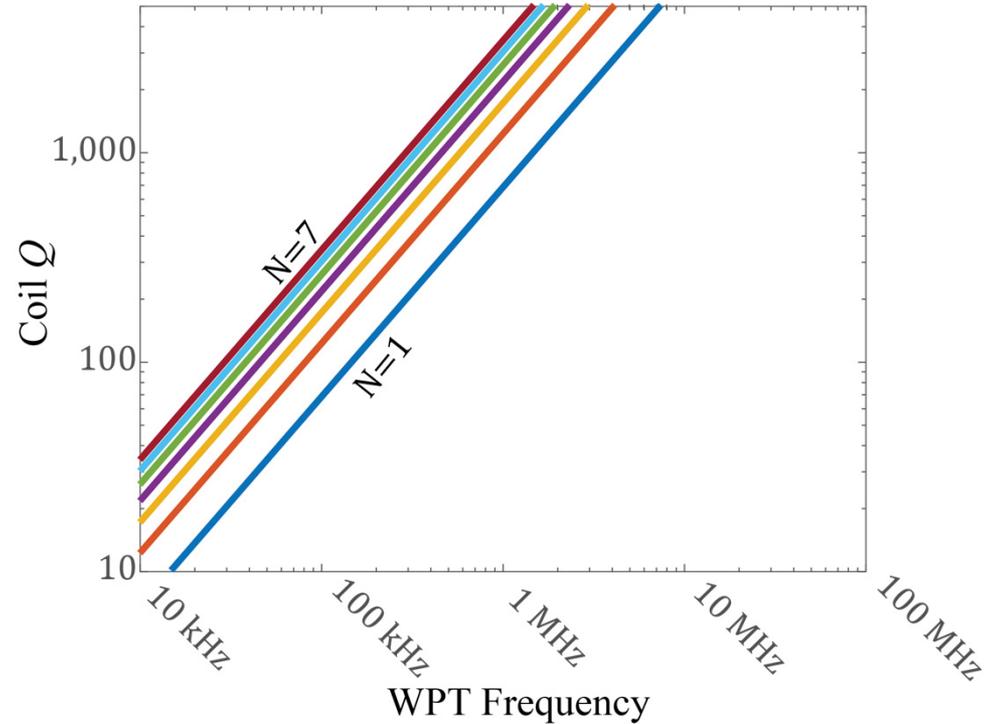
# Increasing $Q$

Example case:

- 500 mm coil diameter
- 16 AWG solid wire
- Proximity loss not considered
- No core material considered

$$Q = \frac{\omega L}{R}$$

- At low frequency,  $R \approx R_{dc}$



$$L = \frac{\mu_0 N^2 (d_{out} + d_{in})}{2} \left( \ln \left( \frac{2.46}{p} \right) + 0.2p^2 \right)$$

$$p = \frac{(d_{out} - d_{in})}{(d_{out} + d_{in})}$$

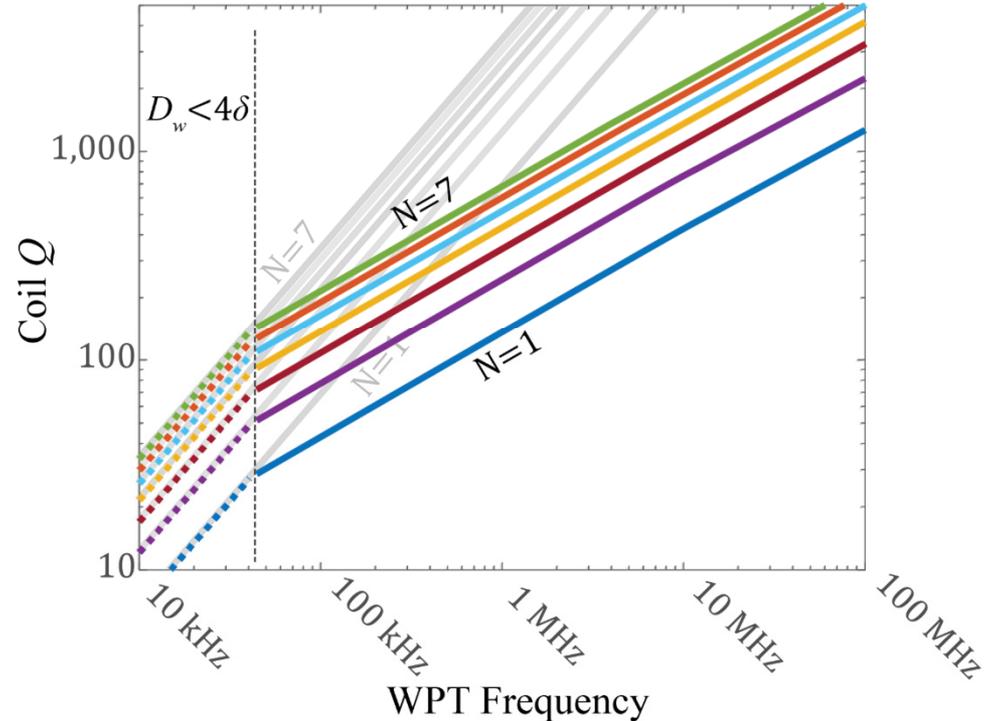
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$$Q = \frac{\omega L}{R}$$

- At low frequency,  $R \approx R_{dc}$
- Skin depth resistance  $R \propto \sqrt{\omega}$ 
  - when  $D_w \gg \delta$



$$\delta = \sqrt{\frac{2\rho}{\omega\mu_0}}$$

$$R_{skin} \approx \frac{\rho L}{\pi D_w} \sqrt{\frac{\rho\omega\mu_0}{2}}, D_w \gg \delta$$

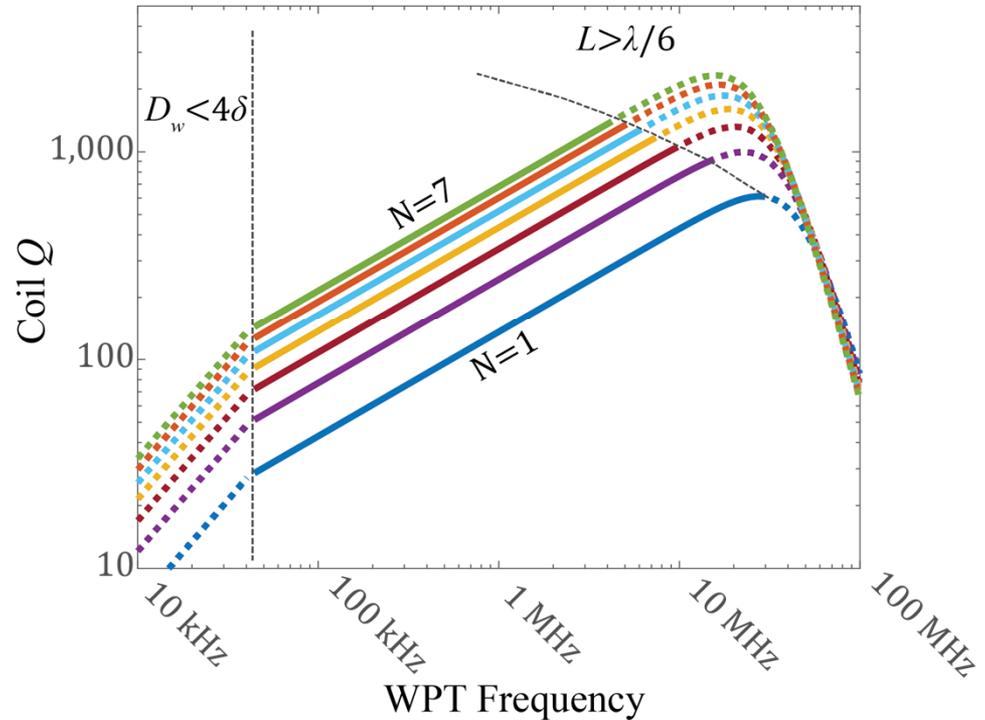
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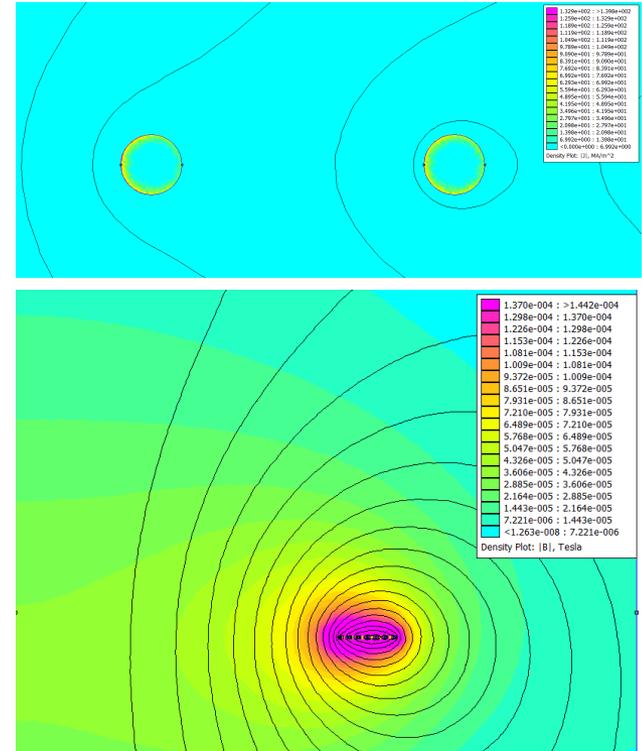
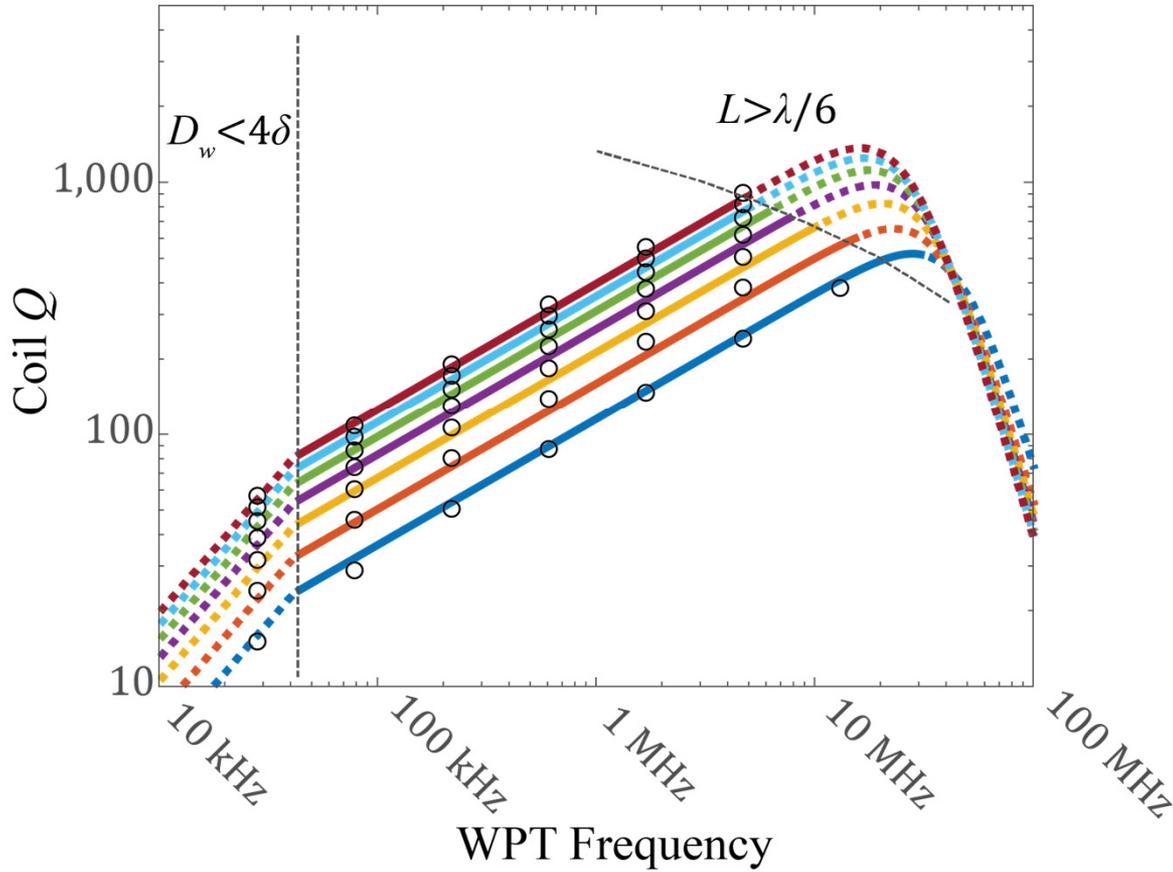
- At low frequency,  $R \approx R_{dc}$
- Skin depth resistance  $R \propto \sqrt{\omega}$ 
  - when  $D_w \gg \delta$
- Radiation resistance  $R \propto \omega^4$ 
  - as  $L \rightarrow \lambda/4$



Other HF limits:

- Parallel capacitance
- Non-coil ac losses

# FEMM Validation



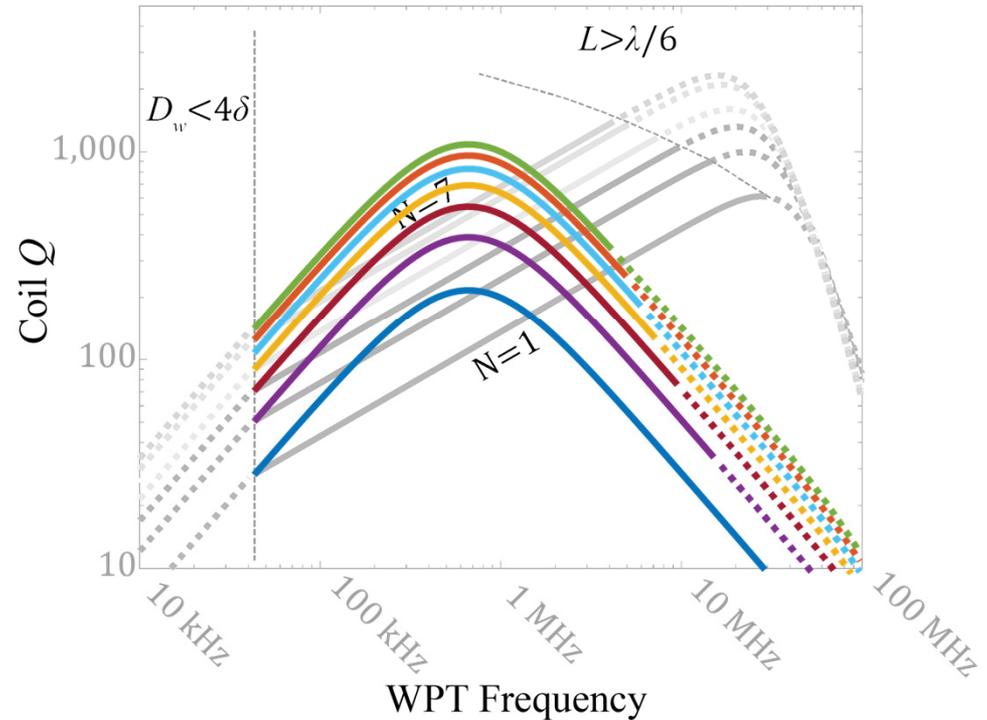
# Litz Wire

Example case:

- 500 mm coil diameter
- 16 AWG solid wire
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- No core material considered
- 45 AWG** strand Litz

$$Q = \frac{\omega L}{R}$$

- At low frequency,  $R \approx R_{dc}$
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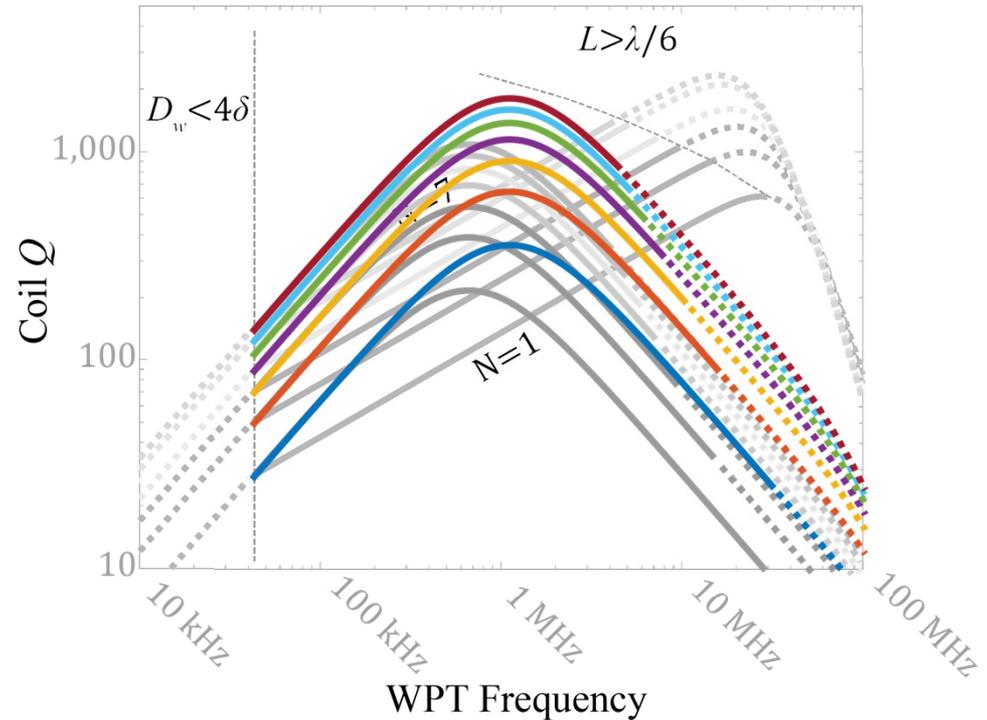
# Litz Wire

Example case:

- 500 mm coil diameter
- 16 AWG solid wire
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- No core material considered
- 45 AWG strand Litz
- 52 AWG** strand Litz

$$Q = \frac{\omega L}{R}$$

- At low frequency,  $R \approx R_{dc}$
- Skin depth resistance  $R \propto \sqrt{\omega}$ 
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- Radiation resistance  $R \propto \omega^4$ 
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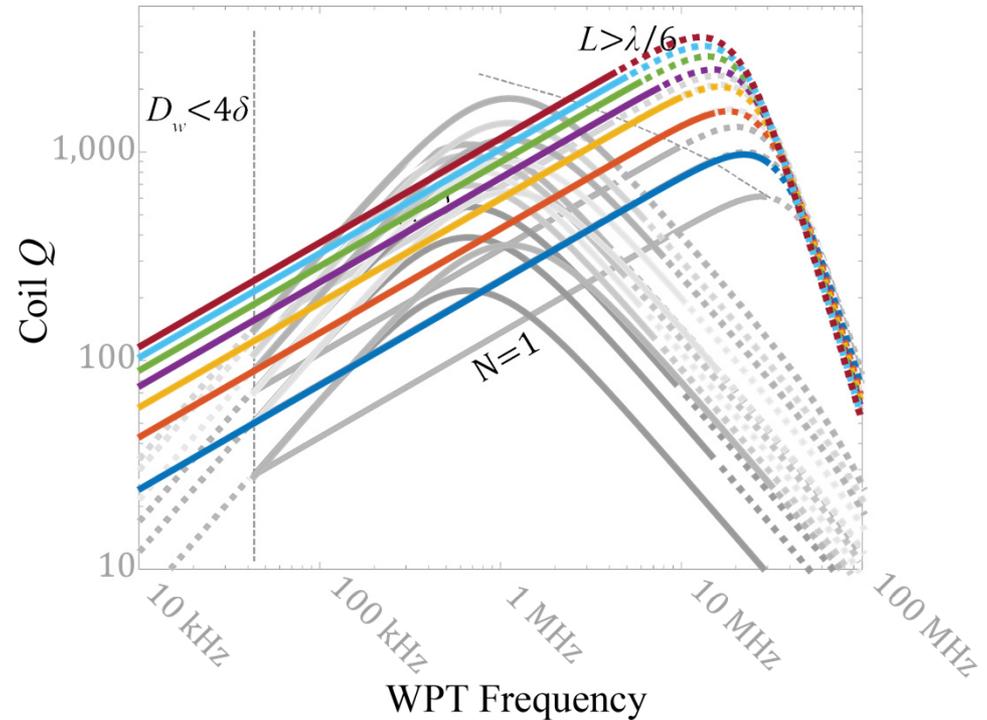
# Litz Wire

Example case:

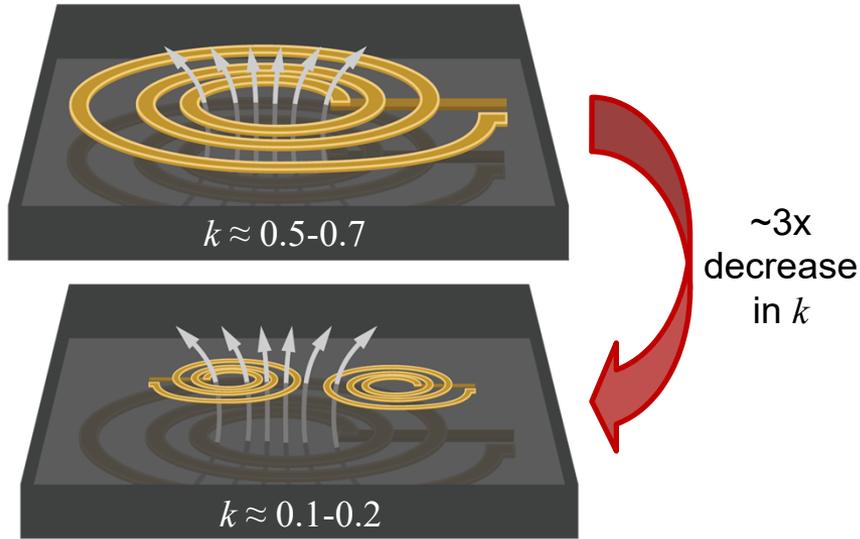
- 500 mm coil diameter
- 16 AWG solid wire
- Proximity loss not considered
- No core material considered
- 45 AWG strand Litz
- 52 AWG strand Litz
- 10 AWG** solid core

$$Q = \frac{\omega L}{R}$$

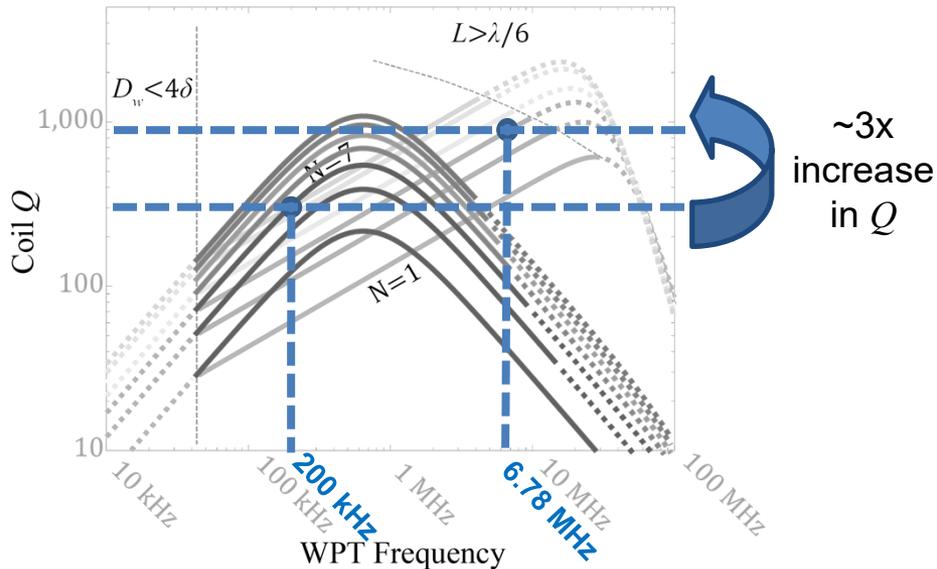
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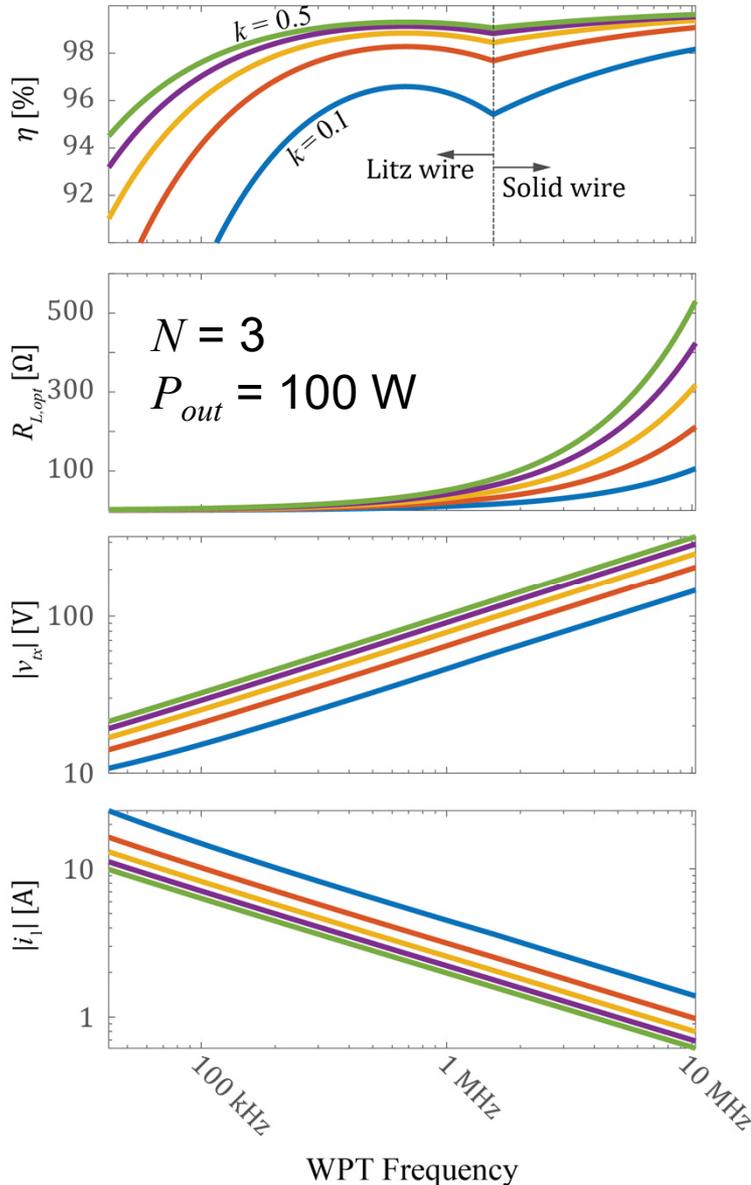
# Frequency Selection: Multi-Receiver WPT



- Higher  $Q$  can offset the reduction in  $\eta$  due to low  $k$

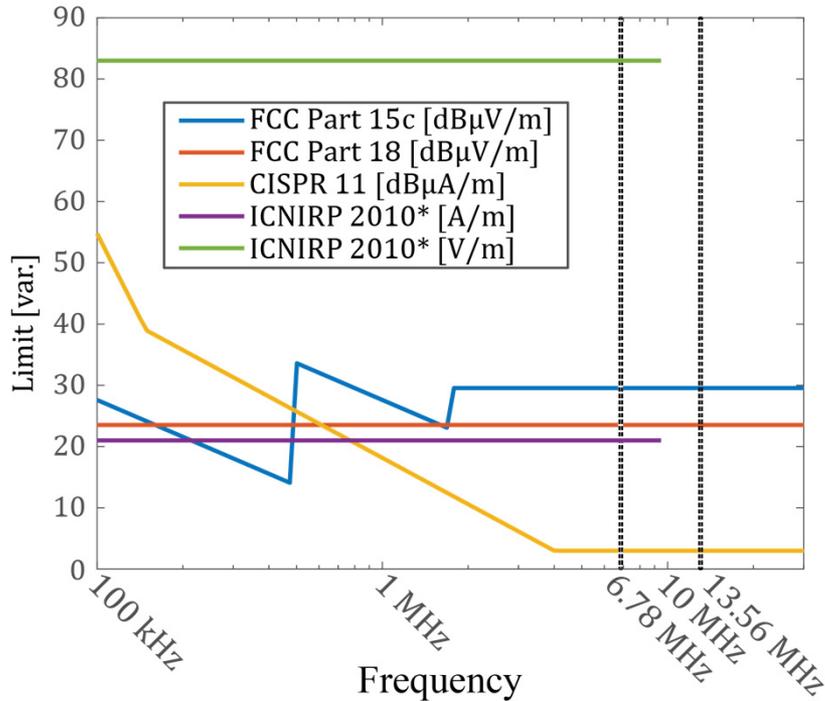


# Frequency Selection: System Implications



- Higher  $Q$  can offset the reduction in  $\eta$  due to low  $k$
- At high frequency,  $R_{L,opt}$  increases
  - High voltage, low current may better-suit power electronics
  - Requires fast-switching, high voltage power devices

# Frequency Selection: Compliance

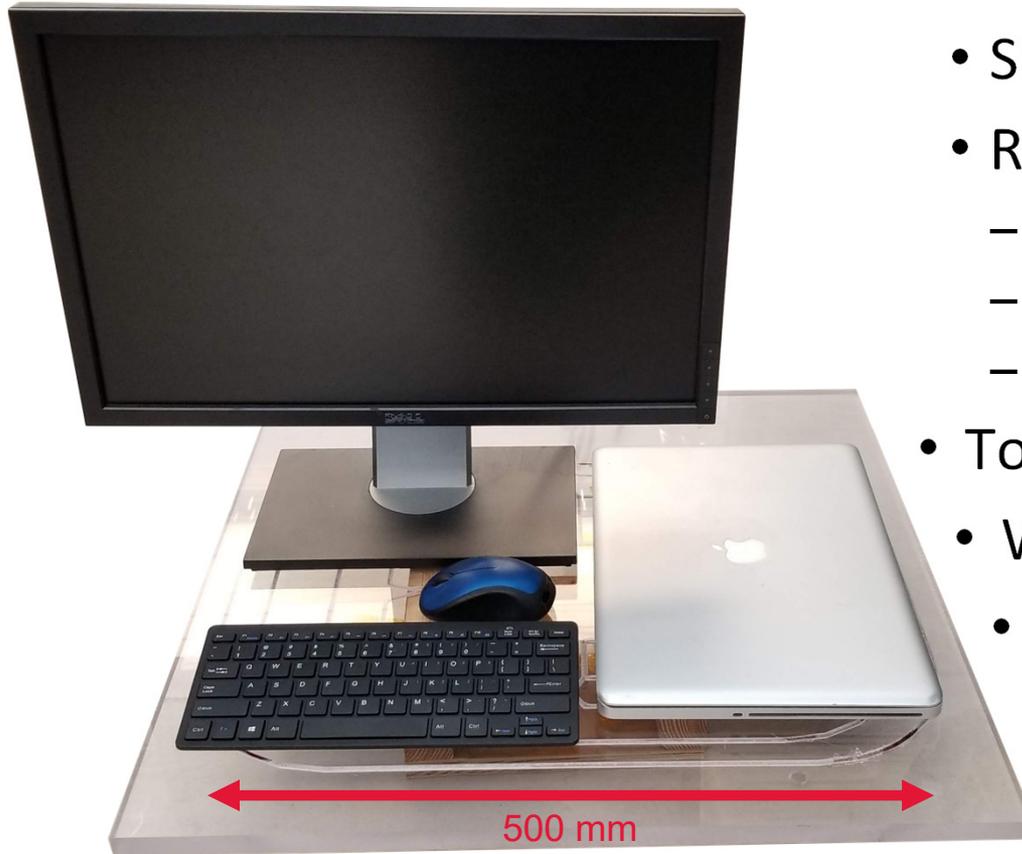


\* At frequencies above 100 kHz, SAR limits need to be considered

- Higher  $Q$  can offset the reduction in  $\eta$  due to low  $k$
- At high frequency,  $R_{L,opt}$  increases
  - High voltage, low current may better-suit power electronics
  - Requires fast-switching, high voltage power devices
- Safety and compliance limits favor operation in ISM bands

# Uniform Field Transmitter Coil

# Multi-Receiver WPT: Target System



- Single transmitter
- Receivers:
  - 25 W Monitor
  - 60 W Laptop
  - 10 W Cell phone
  - ½ W Keyboard
  - ½ W Mouse
- Total  $P_{out} \approx 100$  W
- WPT frequency  $f_s = 6.78$  MHz
- Target  $\eta = 90\%$ 
  - ac line to load dc input

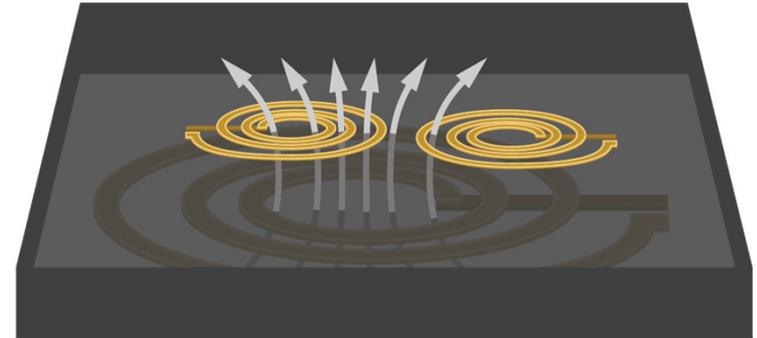
# Transmitter Coil Design

## Issues

1. Field, and therefore coupling varies  $x$ - $y$  position
2. Reflected load alters inverter output current
  - Cross-regulation issue even without cross-coupling

## Approaches

1. Constant-field coil design
2. Constant output current impedance matching network

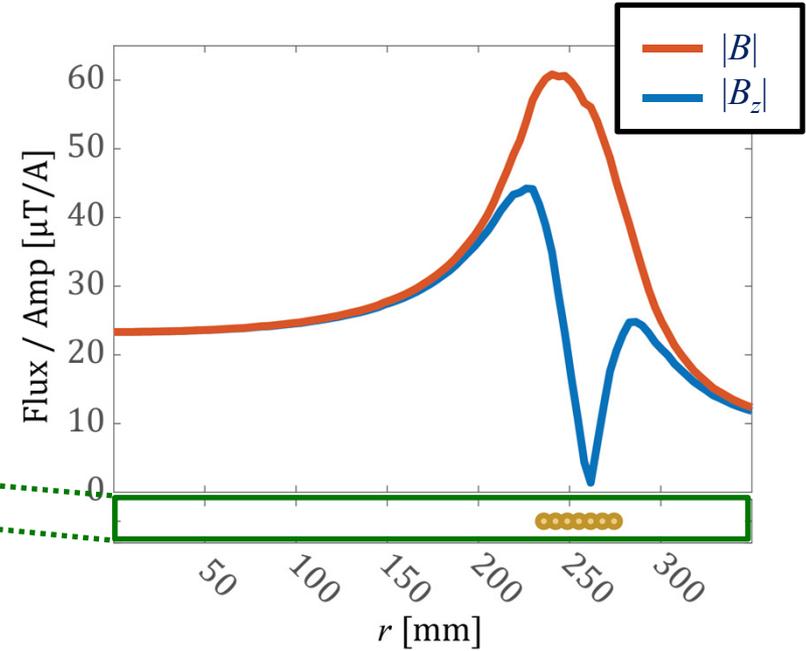
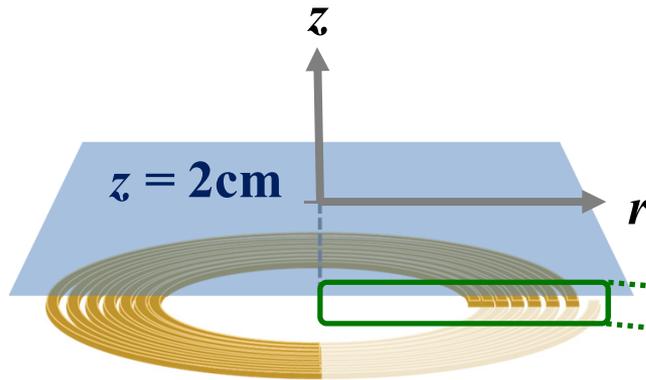


280 mm

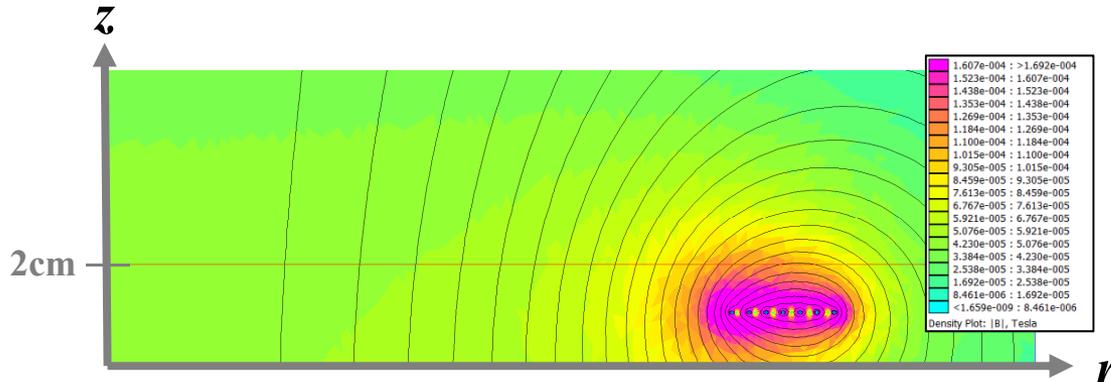


# Coil Flux Distribution

- Flux distribution evaluated in FEMM on receiver plane

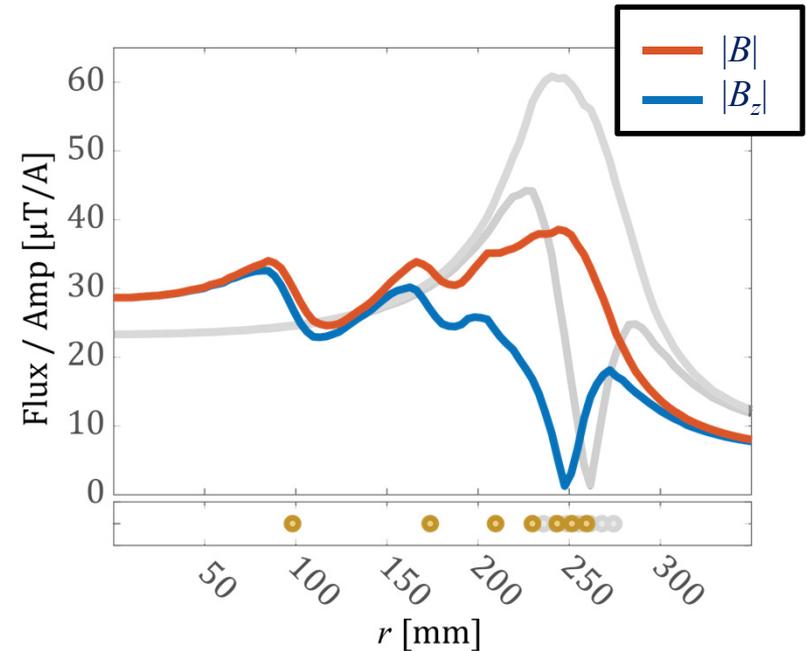


- 2D, axisymmetric simulations of circular coil

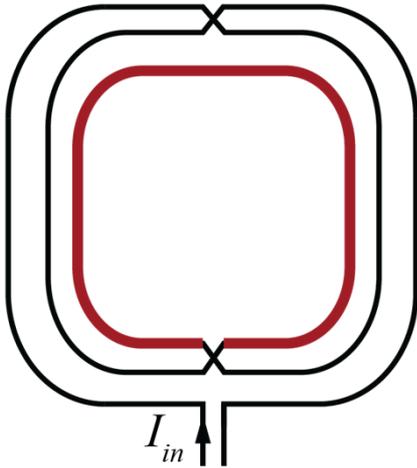


# Coil Flux Distribution

- By changing turn radius, can
  - Reduce variation in  $|B_z|$
  - Reduce peak of  $|B|$
- May reduce  $Q$  if skin effect is dominant
  - Inductance can be regained with additional turns while  $l \ll \lambda/4$
- Finer resolution field shaping possible with more turns

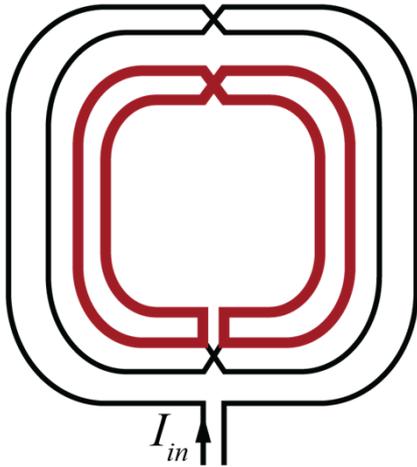


# Symmetric Subturns



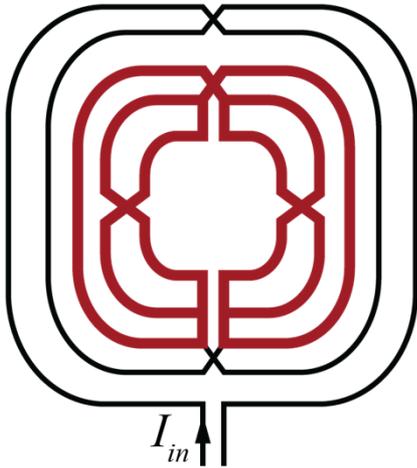
- 3-turn, symmetric coil
  - Current in each turn:  
 $[I_{in} \quad I_{in} \quad I_{in}]$

# Symmetric Subturns



- 3-turn, symmetric coil
  - Current in each turn:
 
$$\left[ I_{in} \quad I_{in} \quad \frac{I_{in}}{2} \quad \frac{I_{in}}{2} \right]$$
- Twisting ensures equal length of each parallel turn
  - Ideally, balanced impedance and equal current sharing

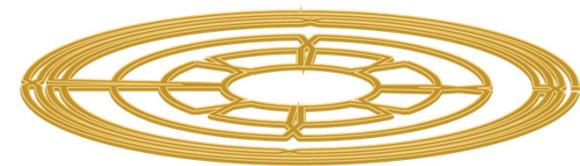
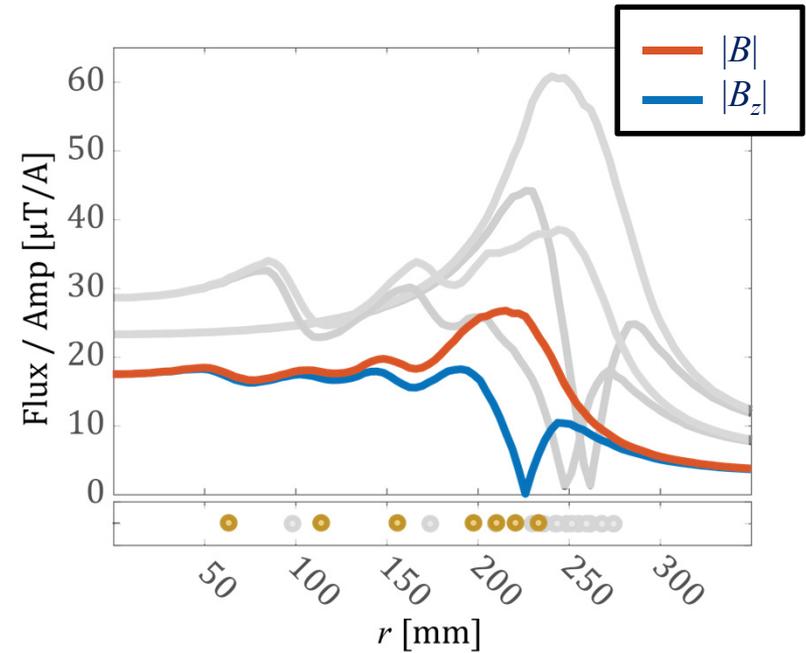
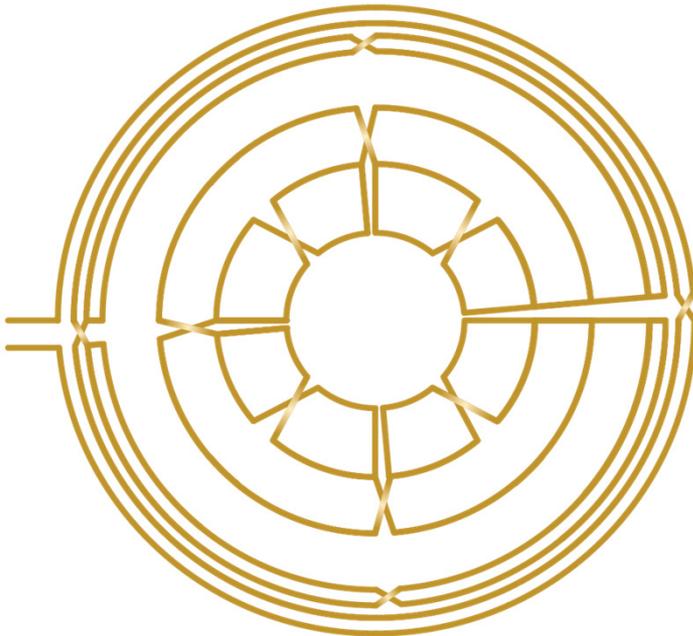
# Symmetric Subturns



- 3-turn, symmetric coil
  - Current in each turn:
 
$$\left[ I_{in} \quad I_{in} \quad \frac{I_{in}}{2} \quad \frac{I_{in}}{4} \quad \frac{I_{in}}{4} \right]$$
- Twisting ensures equal length of each parallel turn
  - Ideally, balanced impedance and equal current sharing
- Reduced inductance compared to single turn at outer radius
- Much shorter effective length than three discrete turns

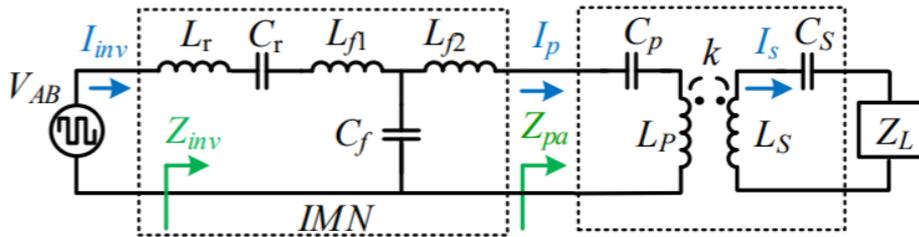
# Coil Flux Distribution

- Fine resolution field shaping possible with paralleled turns
- “Twisting” necessary to balance impedance between parallel turns



# Constant Current IMN

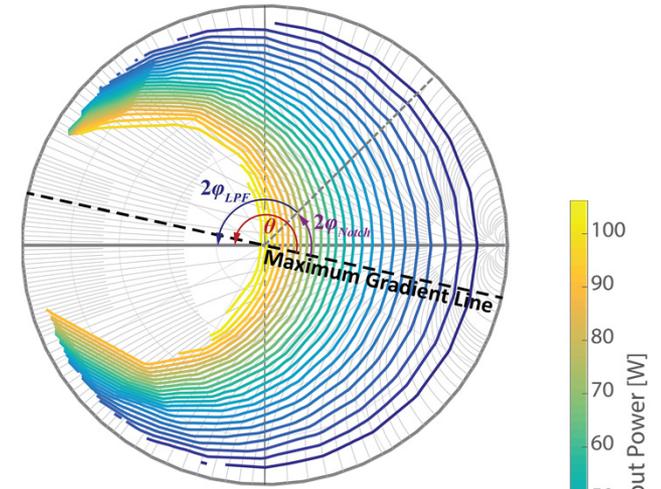
- Impedance matching network between inverter and coils
  - Match reflected load to inverter output characteristic
  - Operate as impedance-admittance converter, i.e. convert output to current source as  $f_s$
  - Additional high frequency attenuation above  $f_s$



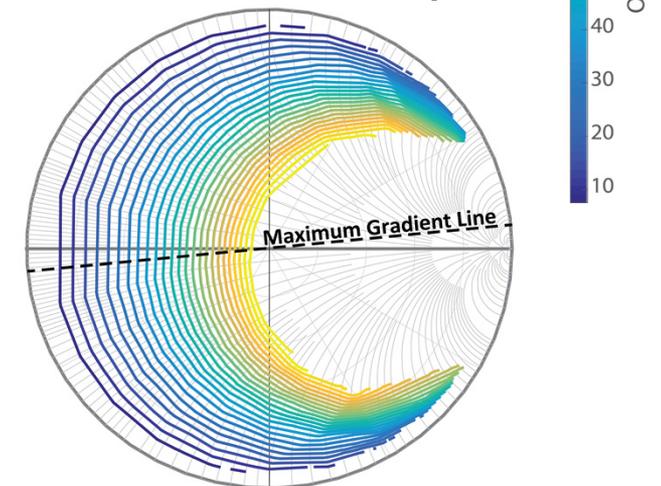
- At  $\omega$  such that  $\omega L_{f1} = (\omega C_f)^{-1}$ , coil current is

$$I_p = \frac{V_{inv}}{\sqrt{2}(j\omega L_{f1})}$$

Independent of loading,  $Z_{pa}$

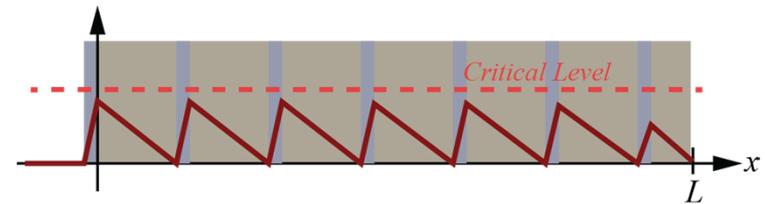
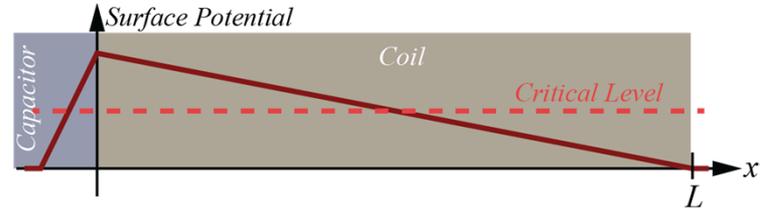
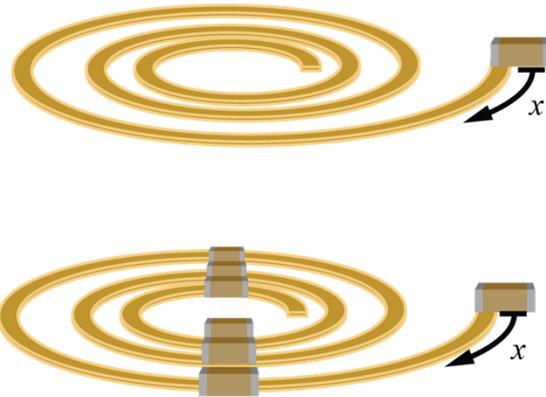


Unfiltered inverter output

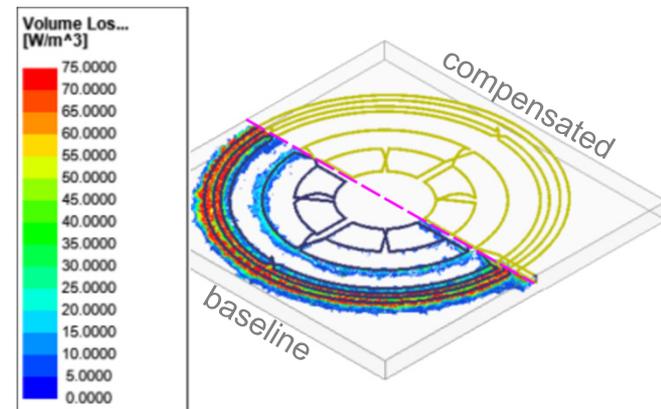


Inverter output with IMN

# Electric Field Compensation

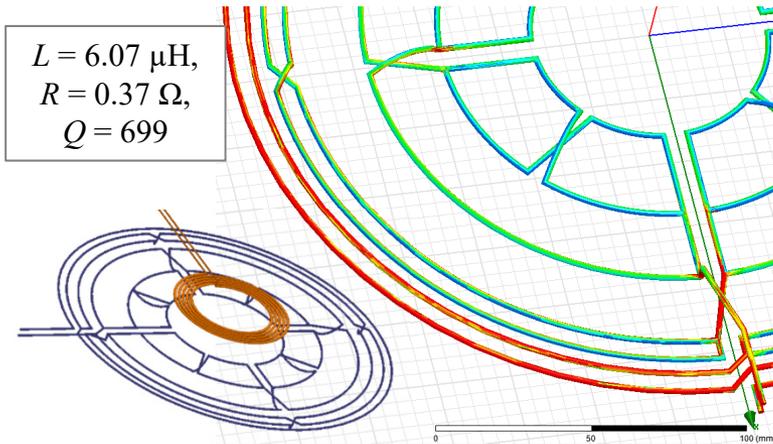
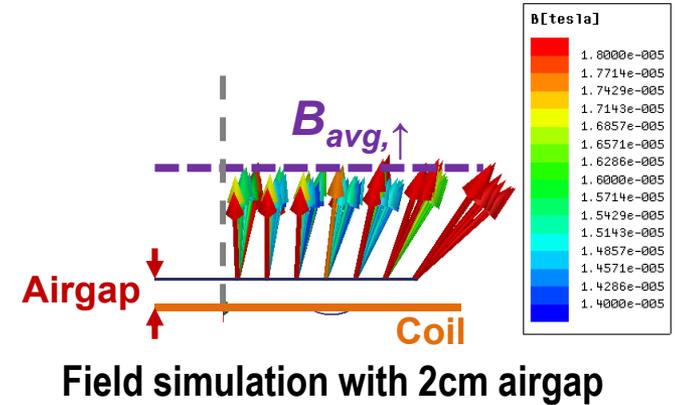


- Longitudinal electric field on coil can cause
  - Additional dielectric loss
  - “Ghost touches”
  - EMI/safety hazard

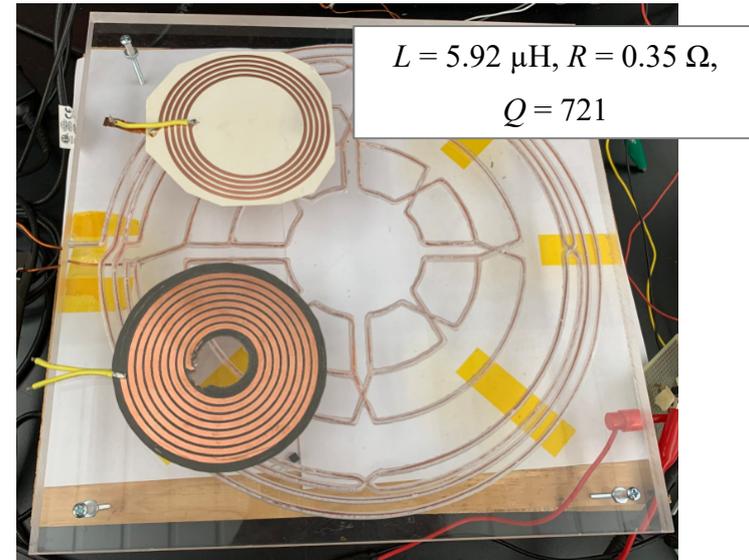


# Coil Prototypes

- Design validated with HFSS simulations
  - Negligible radiated power
  - Current balanced between paralleled turns
  - High- $Q$  at 6.78 MHz
- Peak coil-to-coil efficiency of 95%,  $k < 0.1$

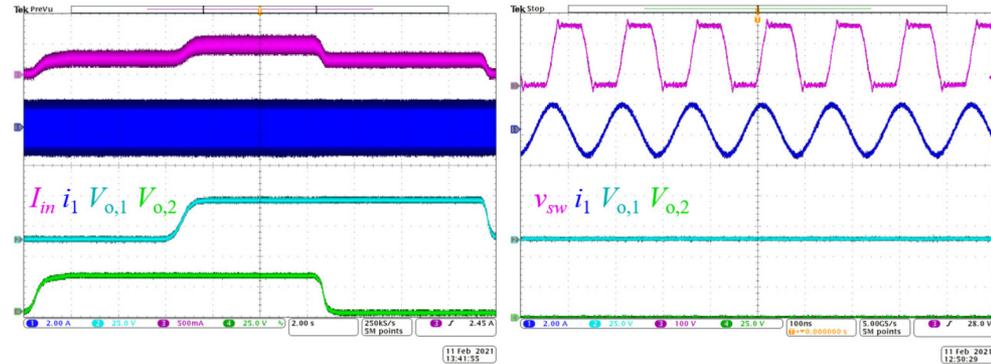
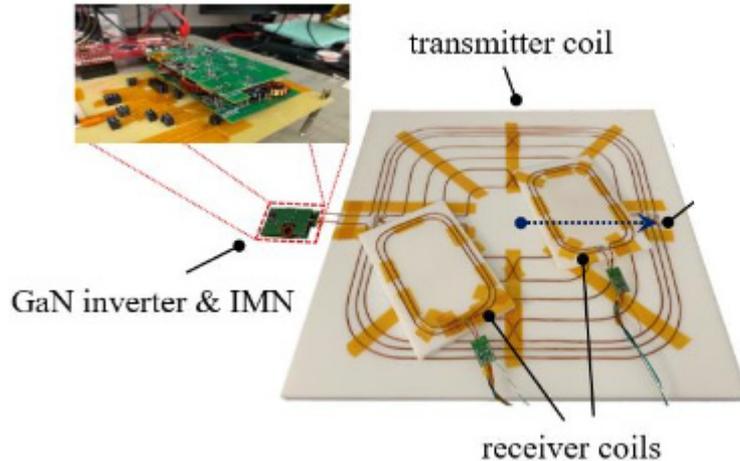


Simulated Tx coil and coil-to-coil model

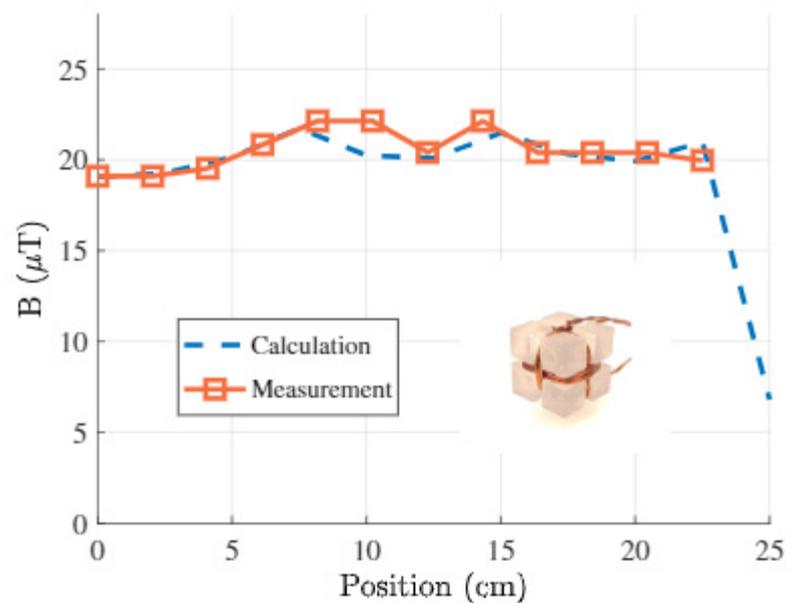


Prototype Tx coil and Rx coils

# Prototype Validation



- Excellent B-field uniformity
- 92.8% dc-dc efficiency at 100W
- >90% ac-to-dc efficiency with PFC

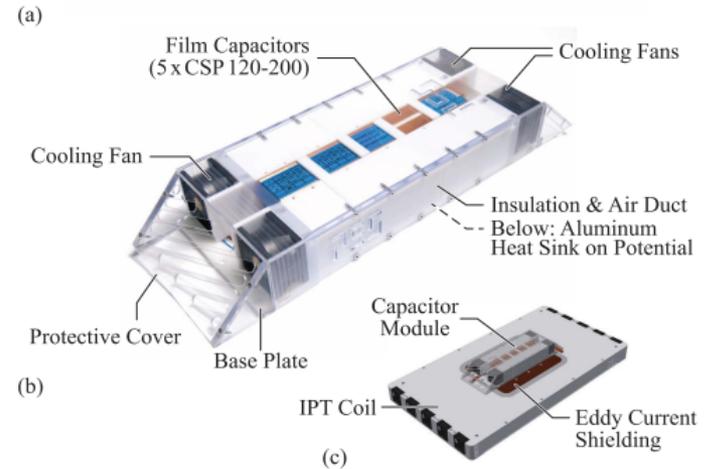
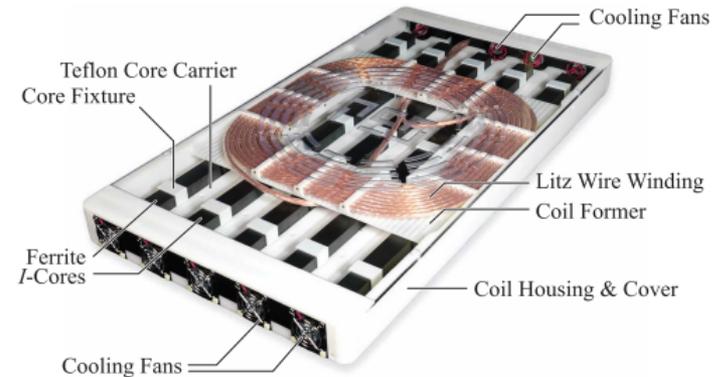


# Series Self-Resonant Coils

# WPT Matching Network

- In low-frequency systems, matching capacitors are large and lossy

Transmitter		Receiver	
MOSFETs		MOSFETs	
162 W	21.2%	109.7 W	20.2%
Capacitor $C_1$		Capacitor $C_2$	
311.5 W	40.7%	223.9 W	41.2%
Litz Wire		Litz Wire	
167.1 W	21.8%	120.1 W	22.1%
Ferrite Cores		Ferrite Cores	
96.1 W	12.6%	69.3 W	12.8%
Shielding		Shielding	
28.3 W	3.7%	20.4 W	3.8%
$\Sigma$ : 764.9 W / 58.5%		$\Sigma$ : 543.4 W / 41.5%	
Total: 1308.3 W / $\eta = 97.45\%$			



(c)

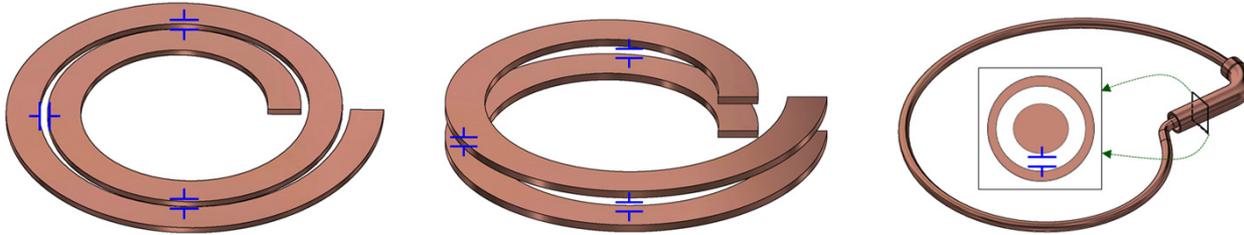
Fig. 8. (a) Photograph of the realized forced-air cooled 50 kW/85 kHz IPT coil prototype (transmitter and receiver coil are identical). (b) Photograph of the resonant capacitor module with forced-air cooling system. (c) Mounting position of the capacitor module at the backside of the IPT coil.

# WPT Matching Network

- In low-frequency systems, matching capacitors are large and lossy
- In high-frequency applications, limited receiver-side volume for high voltage components



# Self-Resonant Coils



- Integrate distributed capacitance into large-area coil
- Controlled parasitic series/shunt capacitance
- Possible to employ capacitive ballasting to control current distribution

# Parallel Resonant SR Coil

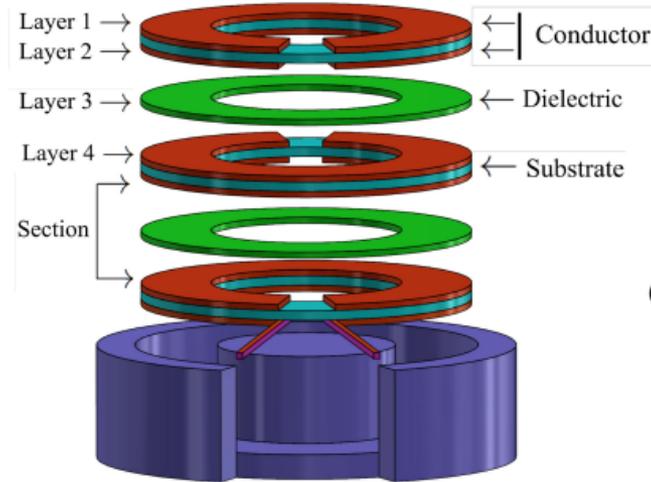
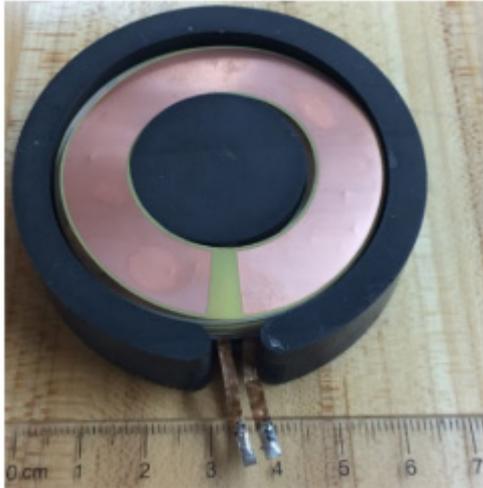
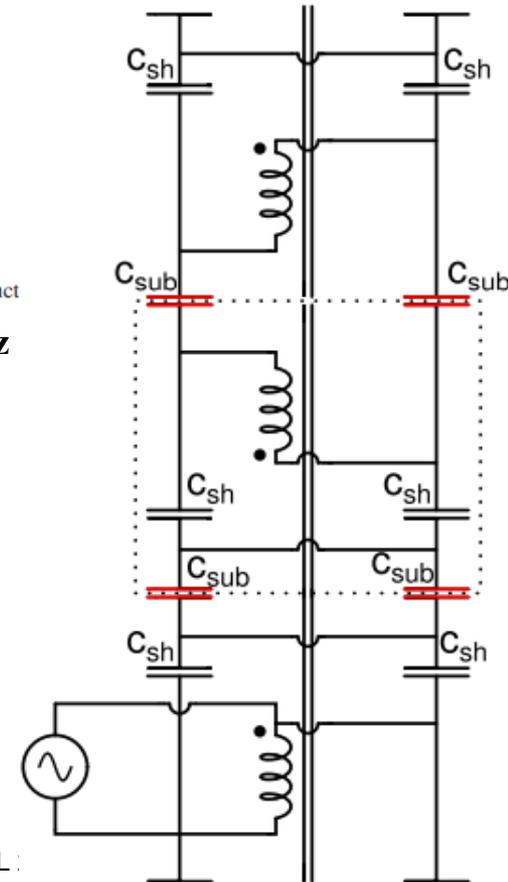


Fig. 7. Layers of a two-section modified self-resonant struct

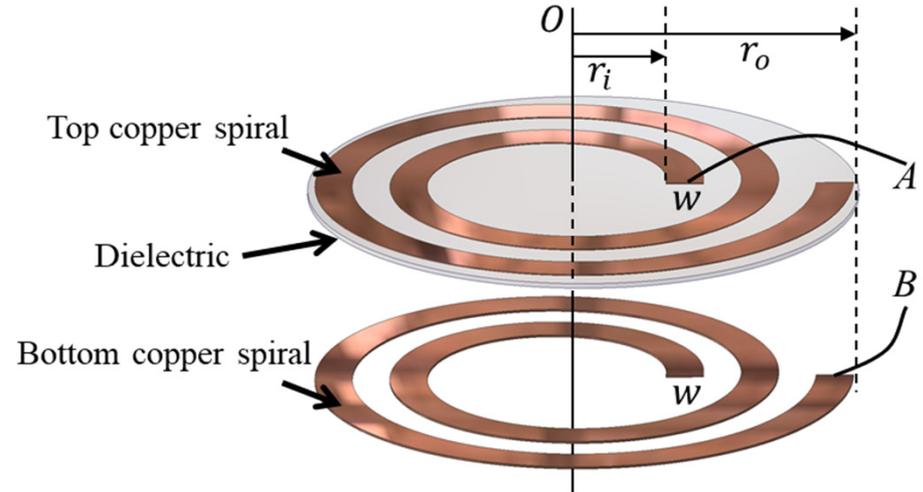
$$L=137\text{nH}, Q=1183, f_o=7\text{MHz}$$

- Parallel resonant coil
- Uses 96 single-turn foil windings with  $D \approx 10\mu\text{m} \ll \delta$
- Highest  $Q$  reported in literature for comparable application
- Requires additional impedance matching

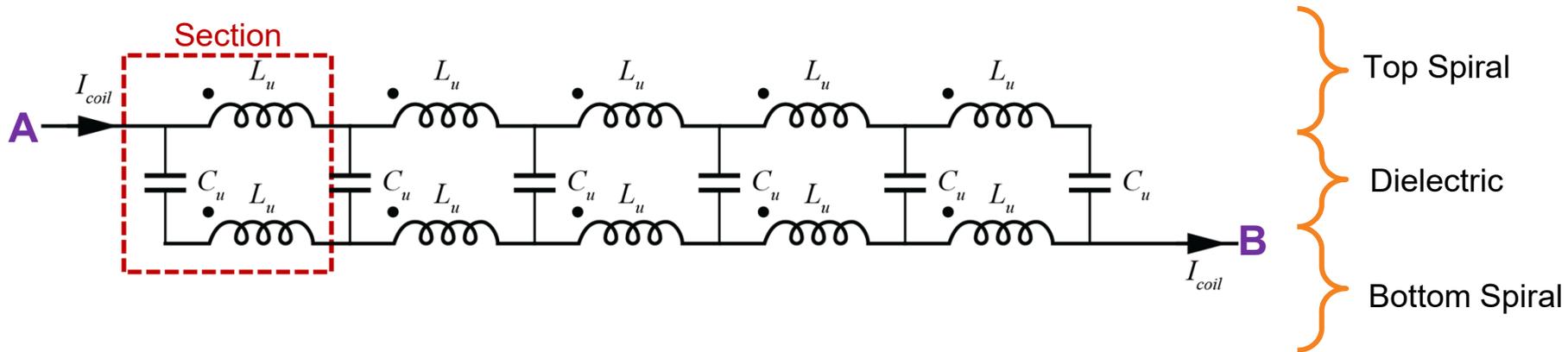


# Series Self-Resonant Coil Structure

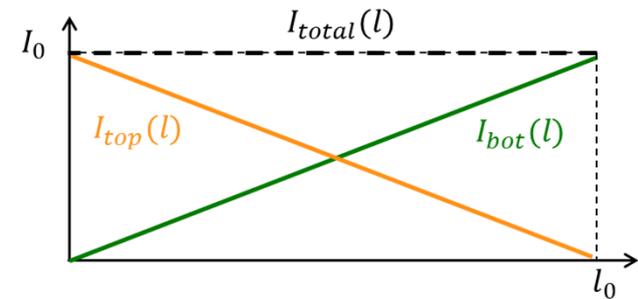
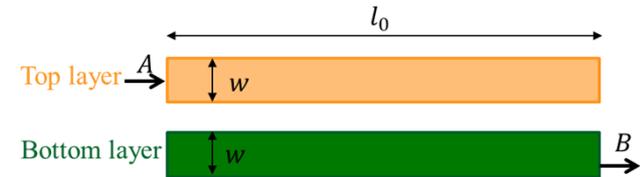
- Multiturn, 2-layer structure
- Low  $\tan\delta$  dielectric material used as substrate
- Geometric design determines electrical parameters



# Lumped Element Model



- For identical  $L_u$ ,  $C_u$  current transfers uniformly over the length of the spiral
- Every current path traverses the spiral once and crosses dielectric once
  - Series  $L$ - $C$  impedance



# Electrical Model

- Inductance



$$L = \frac{\mu_0 N^2 (d_{out} + d_{in})}{2} \left( \ln \left( \frac{2.46}{f} \right) + 0.2 f^2 \right)$$

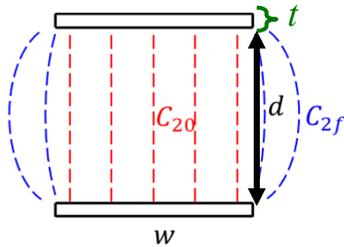
# Electrical Model

- Inductance



$$L = \frac{\mu_0 N^2 (d_{out} + d_{in})}{2} \left( \ln \left( \frac{2.46}{p} \right) + 0.2p^2 \right)$$

- Capacitance



$$C = \frac{\epsilon_r \epsilon_0 \pi w N (d_{out} + d_{in})}{2d} \left[ 1 + \frac{d}{\pi w} \ln \frac{2d}{\pi w} + \frac{d}{\pi w} \ln \left( 1 + \frac{2d}{\pi w} + 2 \sqrt{\frac{t}{d} + \left( \frac{t}{d} \right)^2} \right) \right]$$

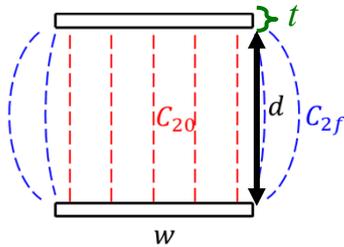
# Electrical Model

- Inductance



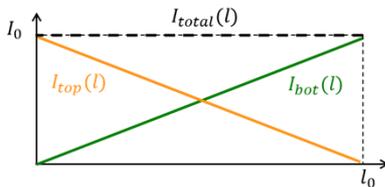
$$L = \frac{\mu_0 N^2 (d_{out} + d_{in})}{2} \left( \ln \left( \frac{2.46}{p} \right) + 0.2p^2 \right)$$

- Capacitance



$$C = \frac{\epsilon_r \epsilon_0 \pi w N (d_{out} + d_{in})}{2d} \left[ 1 + \frac{d}{\pi w} \ln \frac{2d}{\pi w} + \frac{d}{\pi w} \ln \left( 1 + \frac{2d}{\pi w} + 2 \sqrt{\frac{t}{d} + \left( \frac{t}{d} \right)^2} \right) \right]$$

- Resistance

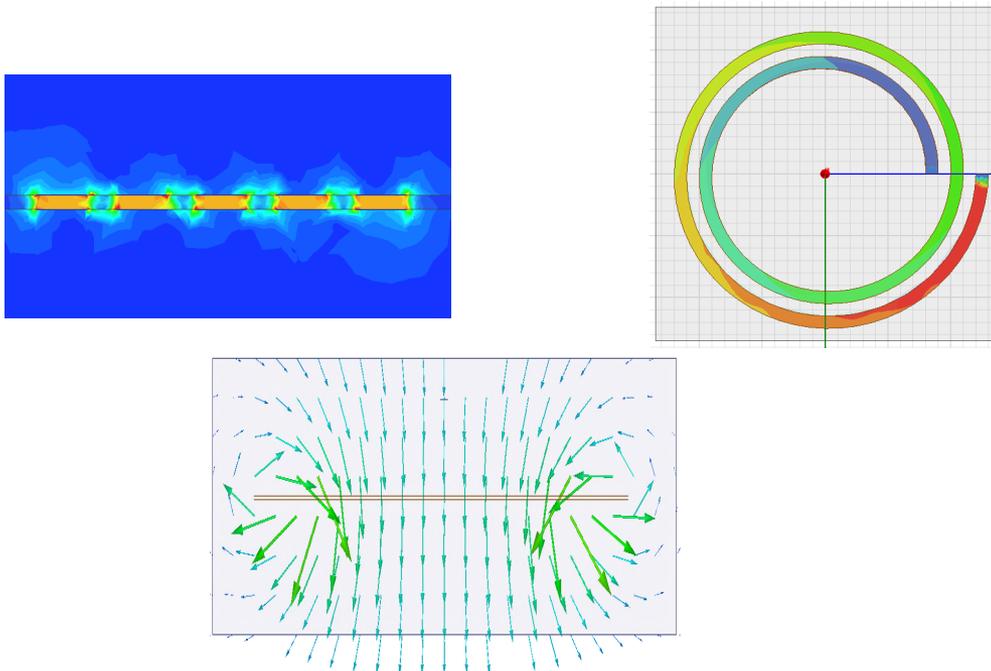


$$R = \frac{2}{3} R_{sprl} + R_{cap}$$

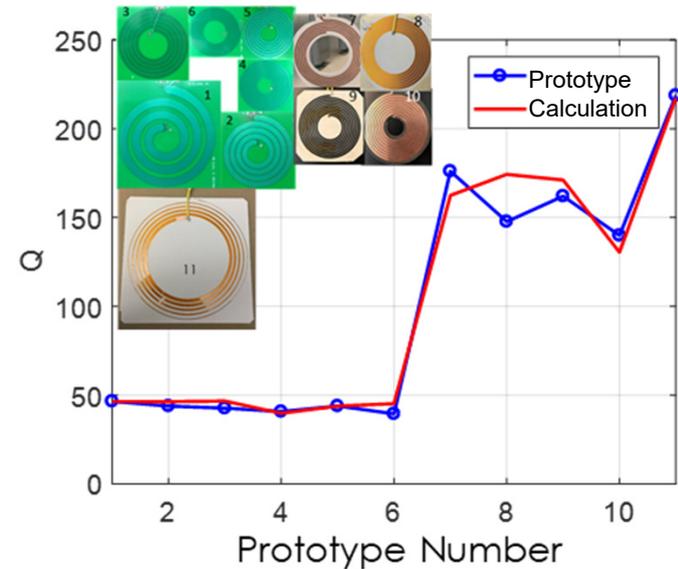
$$= \frac{\rho \pi N (d_{out} + d_{in})}{2wt} \left( \frac{2t}{\delta (1 - e^{-\frac{t}{\delta}})} - \frac{4}{3} \right) + \frac{D_k}{\omega C}$$

# Series Self-Resonant Receiver Coil

- Axially aligned planar spiral windings with internal dielectric used to generate series self-resonance
- Geometrical optimization required to achieve designed reactance while maximizing  $Q$



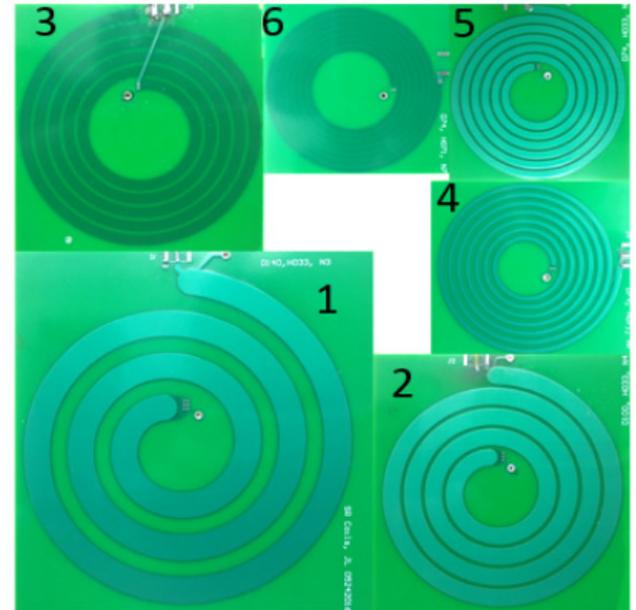
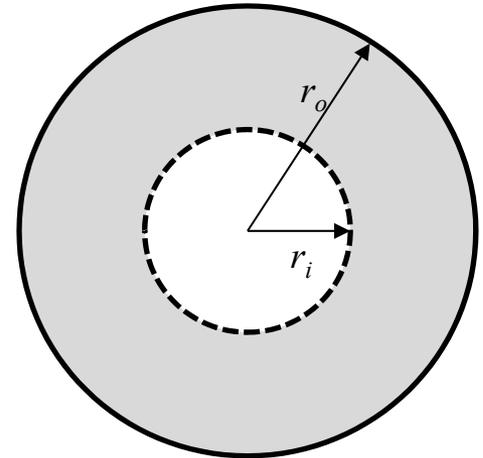
**FEA of Coil E-field, H-field, and current density**



**Receiver coil model validation**

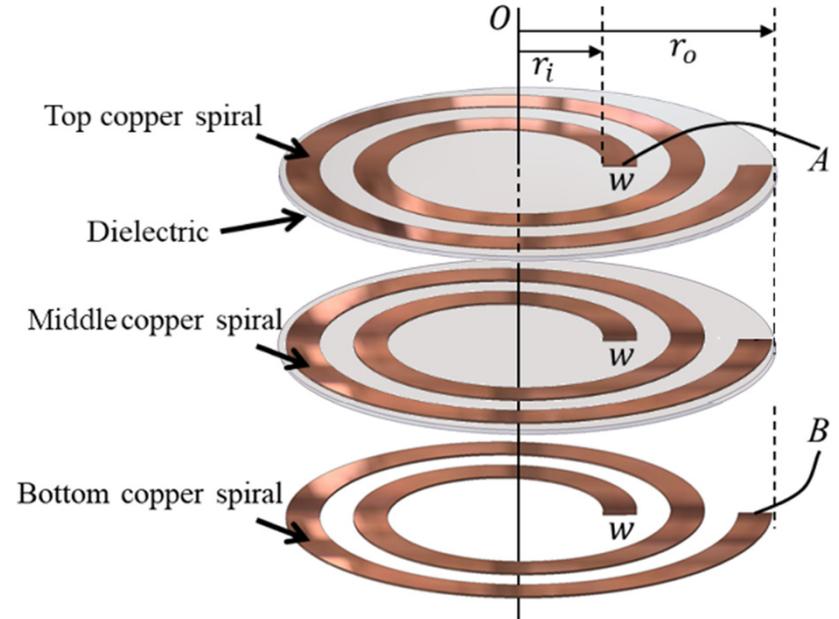
# Performance Constraints

- For a given coil size and dielectric, optimal design is coupled
- Example:
  - Wider traces ( $w$ )
    - ↓
  - Reduces resistance
    - ↓
  - Increases capacitance
    - ↓
  - Requires decreased inductance to maintain resonant frequency
    - ↓
  - $Q$  decreases

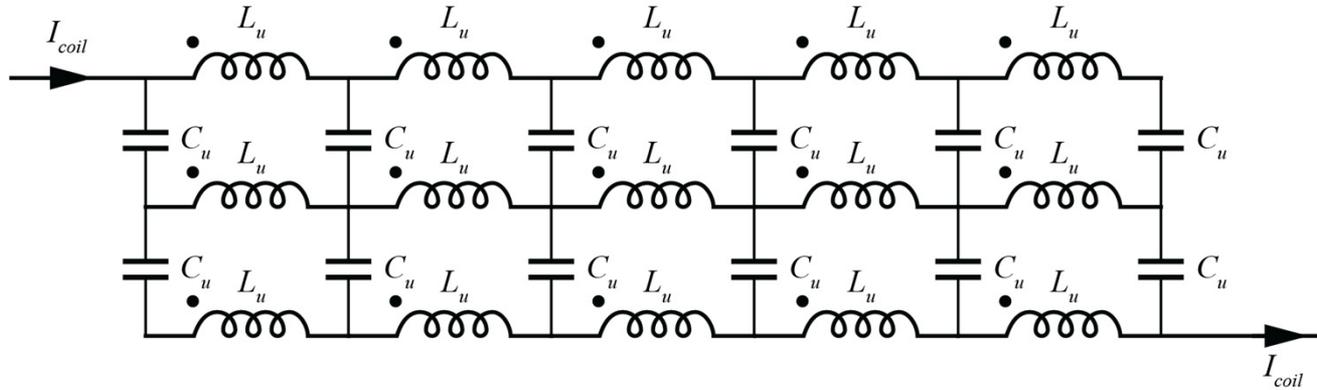


# Multi-layer Series Self-Resonant Coil

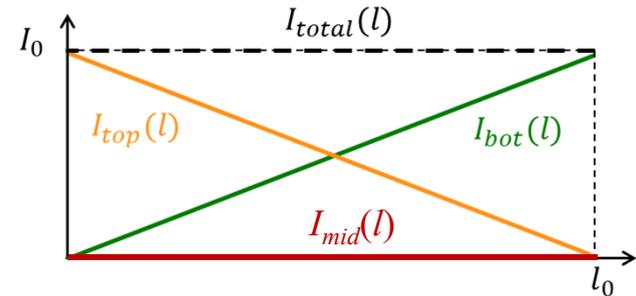
- Goal: use multi-layer series capacitance to allow wider traces, reduce resistance



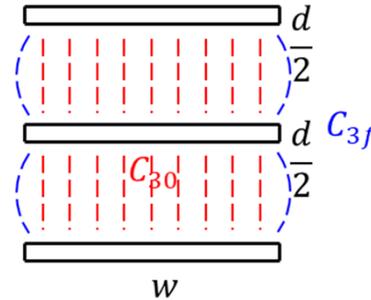
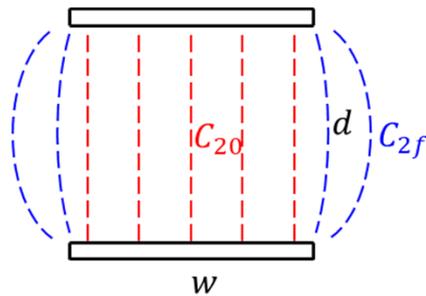
# Initial Attempt



- Uniform width design results in no current sharing in middle layer
- Balanced upper and lower capacitances have equal current
- Surprisingly, still some benefit over 2-layer



# Fringing



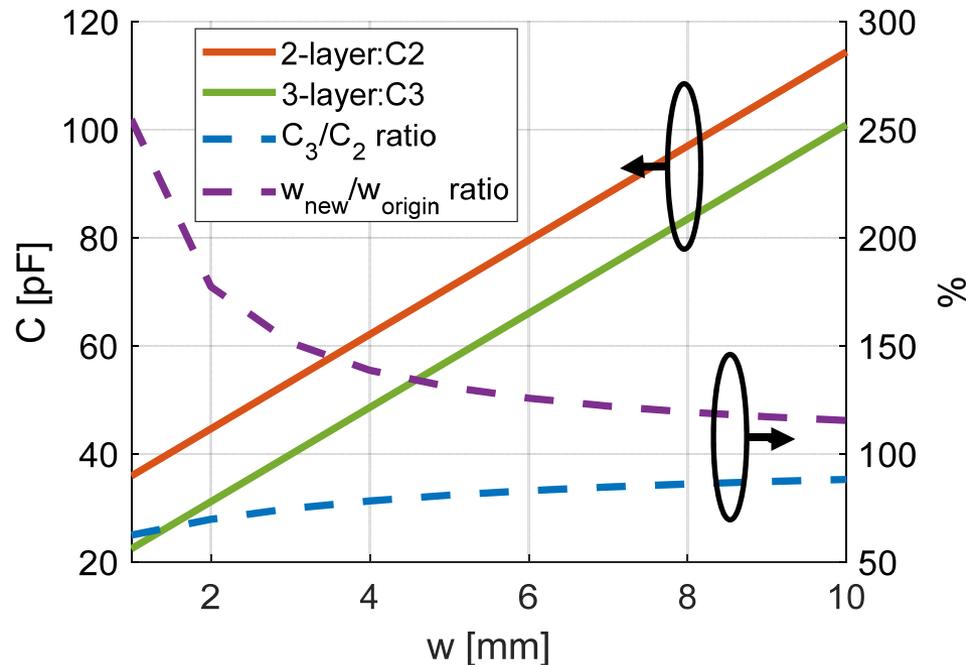
- When  $w \gg d$ , fringing can be neglected
- When  $w \ggg d$ , fringing can dominate parallel-plate capacitance

$$\begin{aligned}
 C_2 &= C_{20} + 2C_{2f} \\
 &= \frac{\epsilon_0 w l}{d} + 2C_{2f}(d)
 \end{aligned}$$

$$\begin{aligned}
 C_3 &= C_{30} + 2C_{3f} \\
 &= \frac{1}{2} \frac{\epsilon_0 w l}{\frac{d}{2}} + 2C_{3f} \left( \frac{d}{2} \right)
 \end{aligned}$$

# Impact on Trace Widths

- Due to reduced fringing, the 3-layer design has smaller capacitance than the 2-layer with the same total thickness
- Discrepancy disappears for  $w \gg d$
- Wider traces can be used to reduce resistance

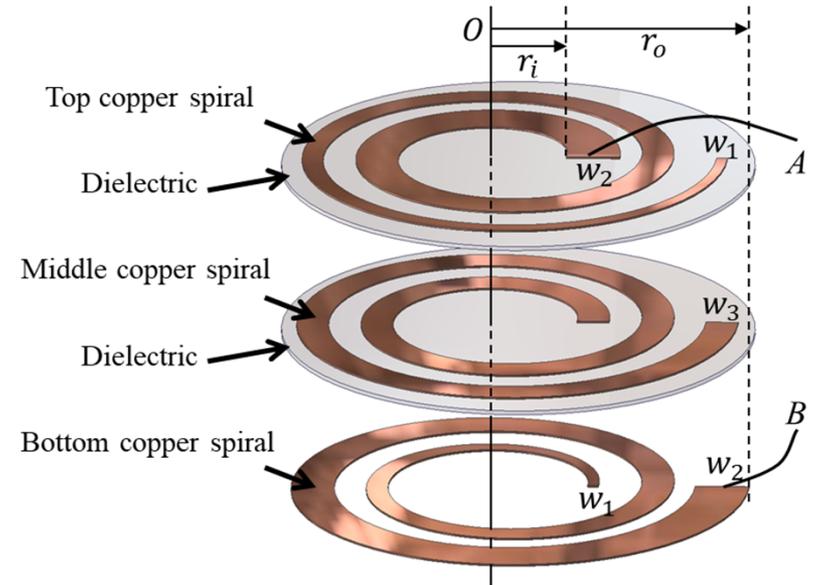


# Multi-layer Non-Uniform Series Self-Resonant Coil

- Modification: Allow variable width traces

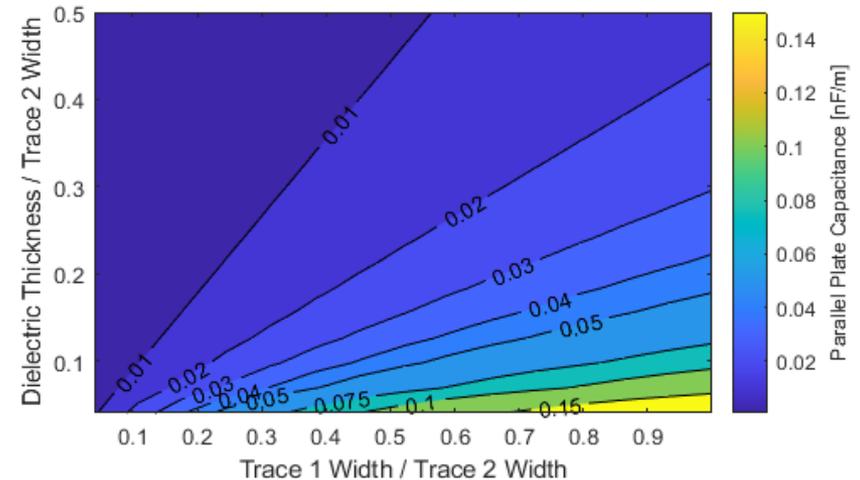
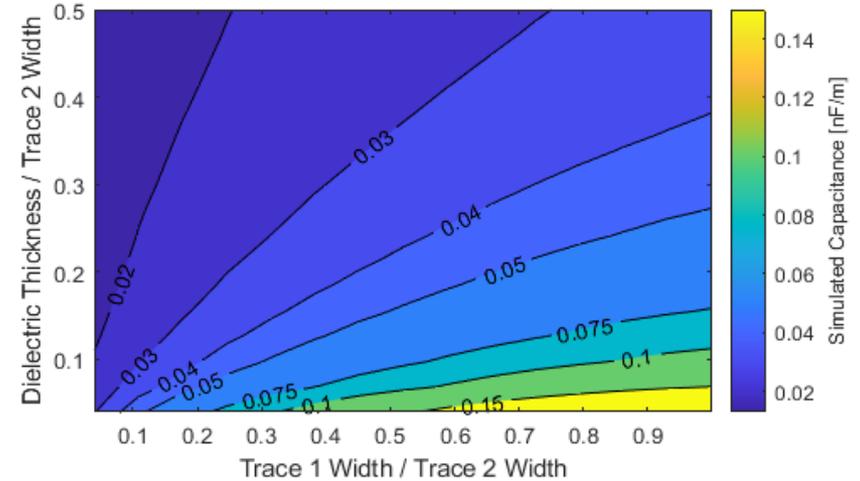
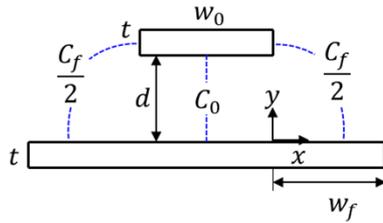
$$w_1 \neq w_2 \neq w_3$$

- Trace profile used to control current transfer to subsequent layers



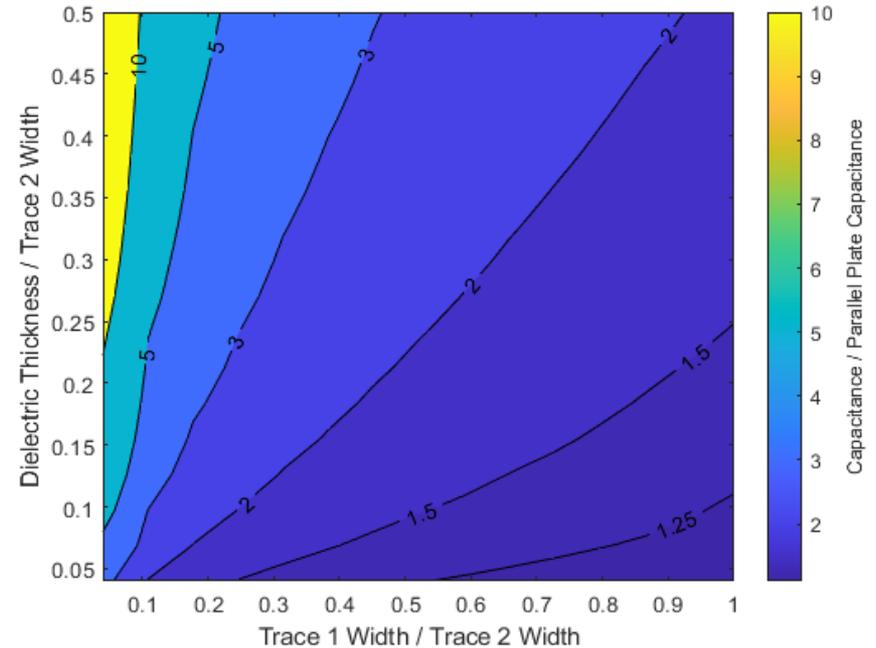
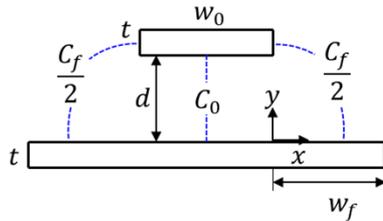
# Nonuniform Trace Fringing

- Fringing effect increases with nonuniform trace width

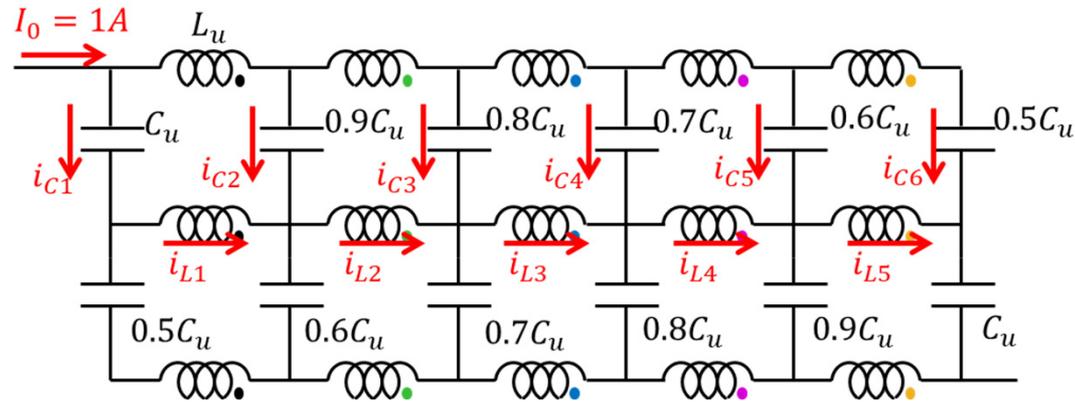


# Nonuniform Trace Fringing

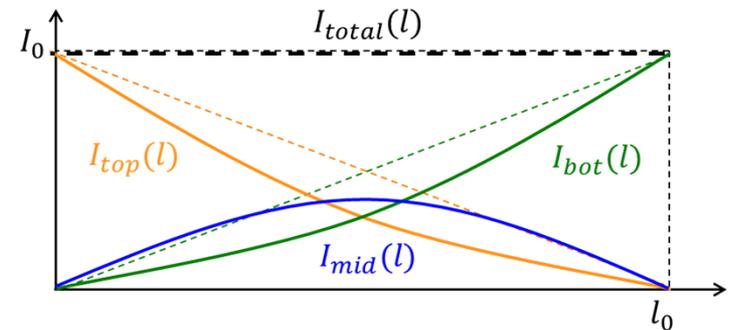
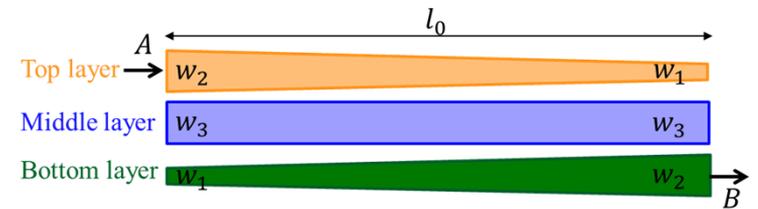
- Fringing effect increases with nonuniform trace width



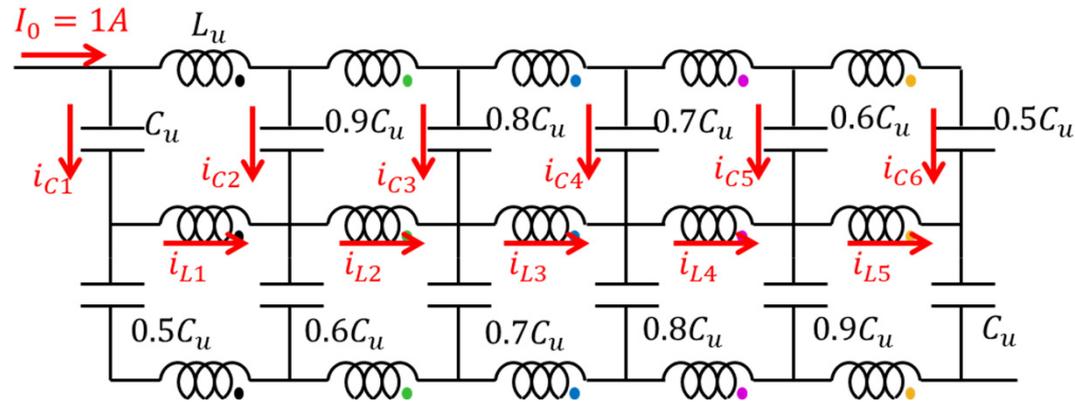
# Lumped Element Model



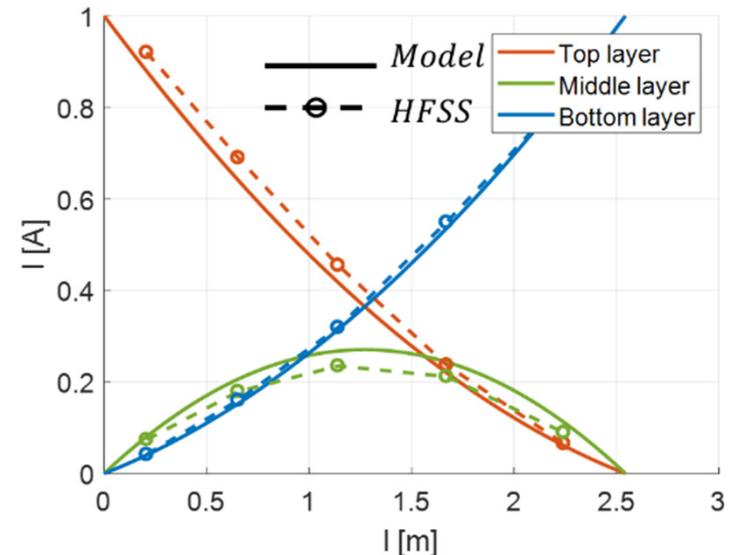
- Differential top-bottom capacitance used to force current through middle layer
- Trace width varies inversely with total current



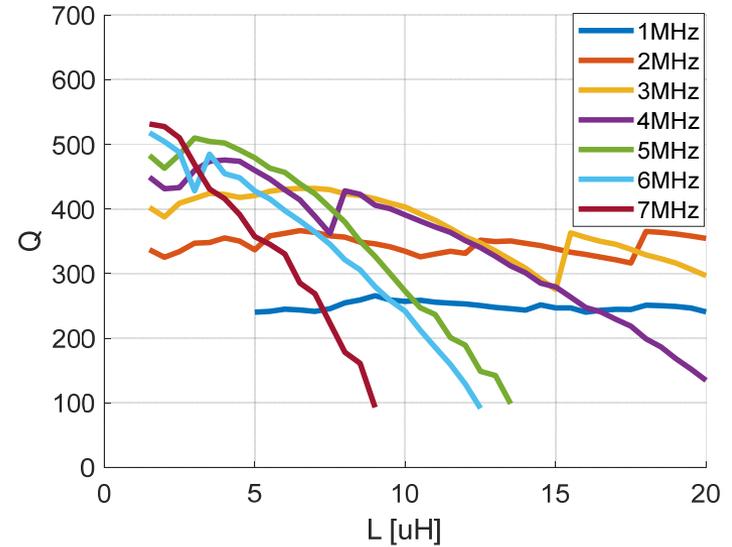
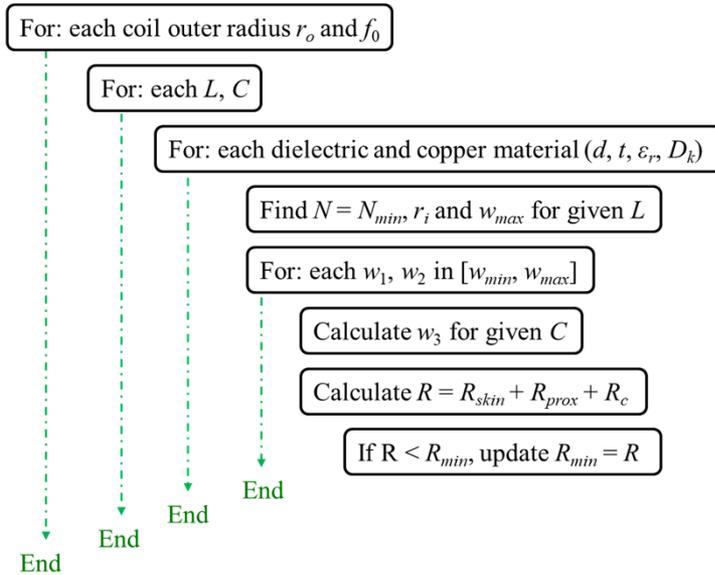
# Lumped Element Model



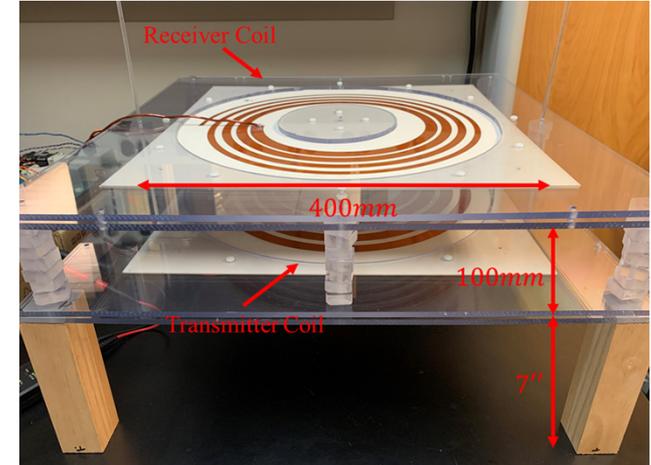
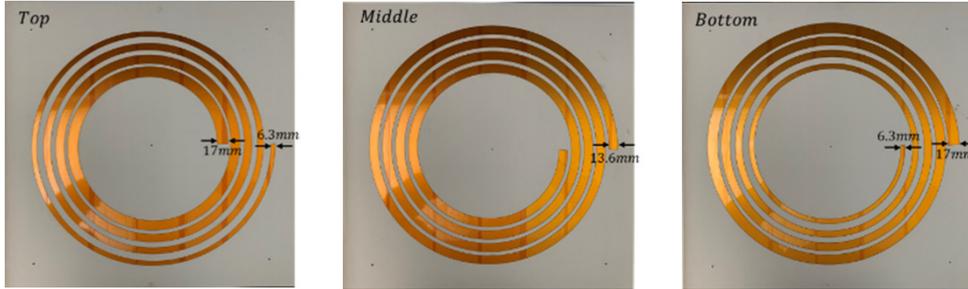
- Differential top-bottom capacitance used to force current through middle layer
- Trace width varies inversely with total current



# Geometric Design



# Prototype Validation



$$L_{tx}=5\mu\text{H}, Q=417, f_o=3\text{MHz}$$

$$L_{rx}=6.5\mu\text{H}, Q=452, f_o=3.2\text{MHz}$$

- Coil design co-optimized with power stage
- High efficiency, 6.6kW 3MHz system
- High- $Q$  without ferrite

